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LABORATORY STUDIES OF THE MOTION OF FREELY FLOATING BODIES IN NON-UNIFORM AND UNIFORM LONG CRESTED WAVES

By

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LIST OF ILLUSTRATIONS

SYMBOLS

A = The waterplane area

B = Beam of the ship

D = Diameter of cylinder

D_v = $\rho g V$ = Buoyancy

H = $2r$ = Wave height

I_y' = $I_y + I_y''$ = Apparent mass moment of inertia with respect to the Y axes where I_y is the actual mass moment of inertia and I_y'' the added moment of inertia

L = Length of the ship

M = Mass

N_1, N_2 = Damping coefficients for heaving and pitching motions

$\overline{M_L G}$ = Metacentric height, longitudinal

T = The period of wave motion

T_1 = The natural period of heaving oscillation

T_2 = The natural period of pitching oscillation

V_z' = $V_z + V_z''$ = Apparent displacement, where V_z is the actual displacement and V_z'' the added displacement

C_L = $\frac{J_L}{BL^3}$ where J_L is the moment of inertia of waterplane area

d = Draft of the ship

g = Acceleration of gravity

r = $1/2\,H$ = 1/2 wave height

w = N/M = Relative damping factor

z = Heave along z-axes

α_w = The waterplane coefficient or the fullness of waterline, defined as the ratio of waterplane area of the ship to the rectangle drawn around this waterplane area

$\mathcal{E}(\lambda/L)$ = Heaving force function

$\mathcal{E}$ = Phase angle

ζ = Amplitude of heaving motion

η = Amplitude of wave motion

Θ = Pitching angle of the ship

$\theta_{(\lambda/L)}$ = Pitching moment function

$K = \frac{2\,w_{1,2}}{\omega_{1,2}}$ = Damping coefficient for heaving or pitching motion

λ = Wave length

$\mu_1,\ \mu_2$ = Amplitude distortion function for heaving and pitching motions

ρ = Density of water

$\omega = \frac{2\pi}{T}$ = Frequency of wave motion

$\omega_1 = \frac{2\pi}{T_1}$ = Natural frequency of the heaving motion

$\omega_2 = \frac{2\pi}{T_2}$ = Natural frequency of the pitching motion

LABORATORY STUDIES OF THE MOTION OF FREELY FLOATING BODIES IN NON-UNIFORM AND UNIFORM LONG CRESTED WAVES.

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ABSTRACT

The motion of freely floating bodies has been studied experimentally in non-uniform wave trains. The data were recorded on movie film and were evaluated for heaving, pitching, drifting and surging motions. In addition to the studies of motion in non-uniform wave trains, a few experiments were made of the motion of freely floating bodies in uniform wave trains and the results compared with a theory developed by Weinblum. A rectangular block approximately 1 foot by $\frac{1}{2}$ foot by $\frac{1}{2}$ foot, a ship model approximately 4 feet by $\frac{1}{2}$ foot with a $\frac{1}{4}$ foot draft, and a cylinder approximately 1 foot long and 1 foot in diameter were used in the experiments which were performed in a wave channel and in a model basin.

I. INTRODUCTION

The motion of freely floating bodies in trains of uniform waves has been studied by many investigators, and in some cases the theoretical results have been compared with experimental measurements (see Bibliography). The agreement seems to be satisfactory when the experimental data were obtained from models in uniform waves under controlled conditions. However, when the theories were applied to the motion of actual ships in a seaway of non-uniform waves the results were not satisfactory(4)*; it appears that the study of floating bodies in uniform waves has more academic value than practical use, and future investigations should be directed toward a solution applicable for non-uniform waves.

The problem is very difficult. First, it is necessary to measure the surface of the sea with all its irregularities, and as waves are often short-crested, the problem must be considered as three-dimensional. Second, it is necessary to determine how a ship, having six degrees of freedom, will act in a seaway. The solution of this problem may be pursued in four different manners:

(1) Solving the problem theoretically.
(2) Using experimental results as obtained under controlled wave and model condition.
(3) Making direct measurements on a prototype under actual conditions.
(4) Statistical observations.

If possible, all four methods should be used and the results compared with each other.

The present studies have been confined to the first two methods, that is, (1) theoretical and (2) experimental - by means of controlled model studies.

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* For numbers in parentheses refer to the Bibliography at the end of the paper.

They are limited to long-crested non-uniform waves, although a few tests were performed to obtain certain information on the motion of bodies in uniform wave trains.

The purpose of this report is primarily to describe the laboratory procedure used in working on the problem, and to show some experimental results of the ship motion studies. The theoretical studies, which have been done simultaneously with the laboratory studies, and their comparison with the laboratory results are presented in a separate report(2).

II. LABORATORY EQUIPMENT AND PROCEDURES

Test Models: In the first experiments, a wooden rectangular block was used (Figure 1). The block was 1 by 0.425 by 0.242 foot. The reason for the rectangular form was to simplify the computation of theoretical characteristics and motions. The block was soaked in water for 24 hours prior to using it in a test, so that no water would be absorbed during the experiment. After soaking, the block was balanced in still water by inserting lead shot in holes drilled along the bottom of the block. Later the holes were sealed with paraffin. After the block was balanced, the draft was measured in still water and was found to be 0.142 ft.

In the later experiments a ship model (the "Clairton") was used (Figure 2). The ship model was nearly wall-sided (except at the ends) and was approximately 4 ft. long, 0.55 feet wide, with a draft of 0.265 ft. The fullness at water line was approximately $\alpha_w = 0.85$*. In Figure 6 the ship model and the rectangular block are shown together for comparison.

In order to get some indication of the heaving motion of a non-wall-sided freely floating body in a seaway, a circular cylinder was built (Figure 3). The cylinder was 1 foot by 1 foot in diameter. Lucite was used so that there would be no problem due to absorption of water during the tests. The cylinder was weighted uniformly around the entire body in such a manner that the draft was equal to the radius of the cylinder. The length of the cylinder was just slightly less than the width of the channel; thus, the body could move freely, but no waves could be generated (due to free oscillation) at the ends of the cylinder and be reflected from the sides of the channel and cause erroneous results. The cylinder was used only for experiments in the wave channel and with uniform waves.

Wave Channel: The first experiments were done in a wave channel 1 foot wide by 3 feet deep by 60 feet long, with one side of the channel constructed of a series of 3 feet by 3 feet glass plates, mounted in steel frames (See Figure 4). Waves were generated by a flapper-type generator, mounted at the one end of the channel. Both the amplitude and the period of the flapper movement were adjustable. The period could be varied between approximately 0.4 seconds and 2 seconds and the wave amplitudes between 0 and 0.5 foot. The water depth during all experiments was 2 feet and so the wave lengths varied between approximately 0.8 foot and 14 feet. At the opposite end of the channel from the wave generator a sloping beach was installed for the purpose of preventing wave reflections.

The motions of the floating bodies were recorded by a 35 mm. movie camera

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* The fullness at waterline, a term used by naval architects, is the ratio of waterplane area of the ship to the rectangle drawn around this waterplane area. It equals 1 for the block, having a rectangular waterplane area.

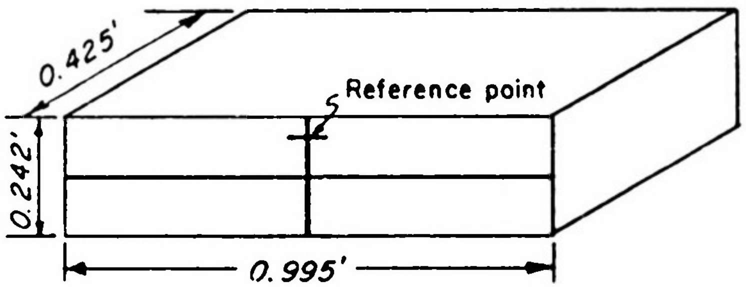

Figure 1. Rectangular Block

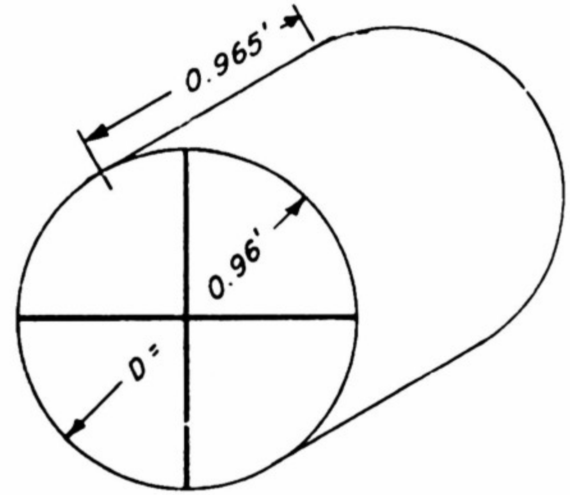

Figure 3. Cylinder

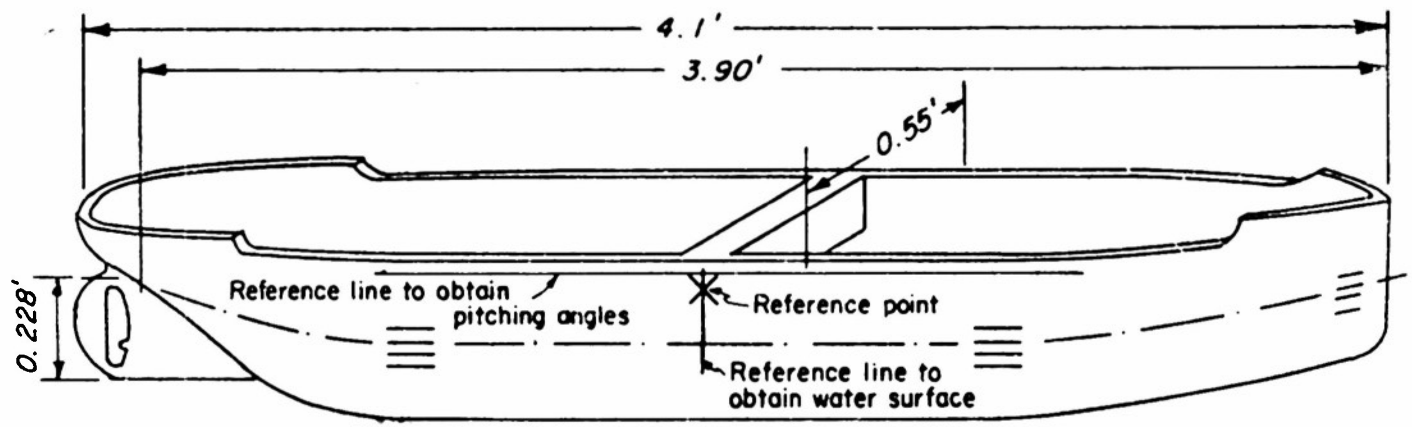

Figure 2. Ship Model

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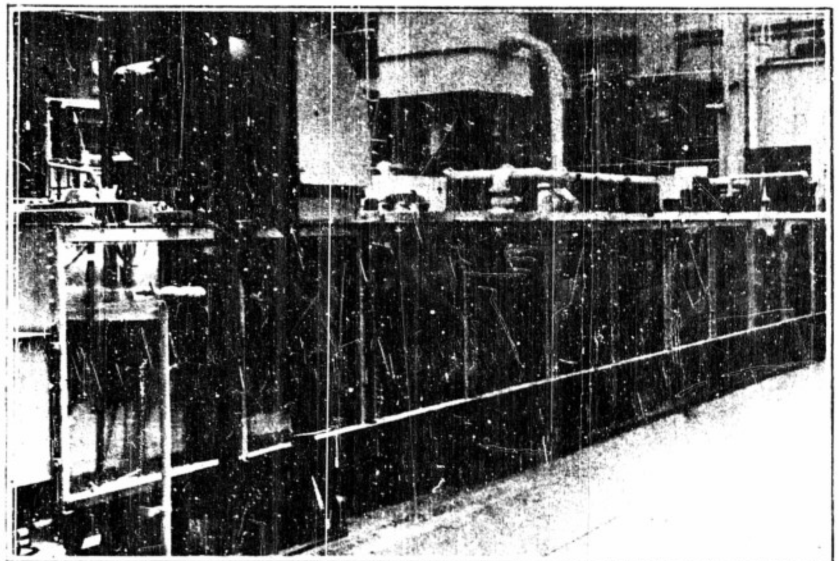

Fig. 4 - Laboratory wave channel

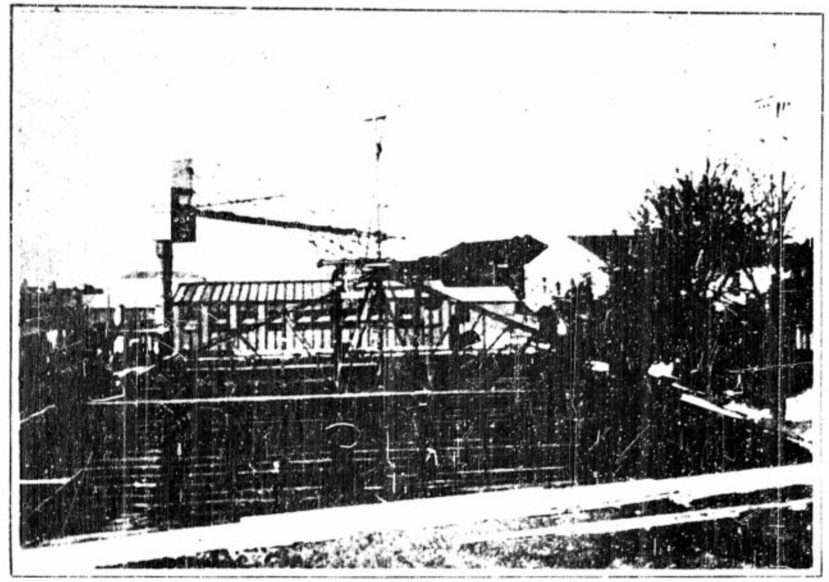

Fig. 5 - Open wave basin

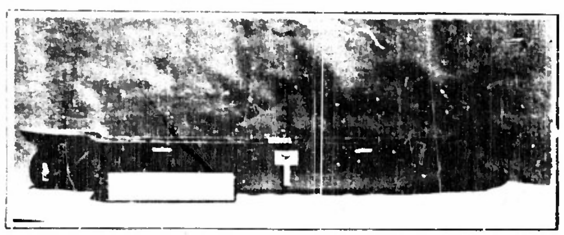

Fig. 6 - Rectangular block and ship model as used in experiments

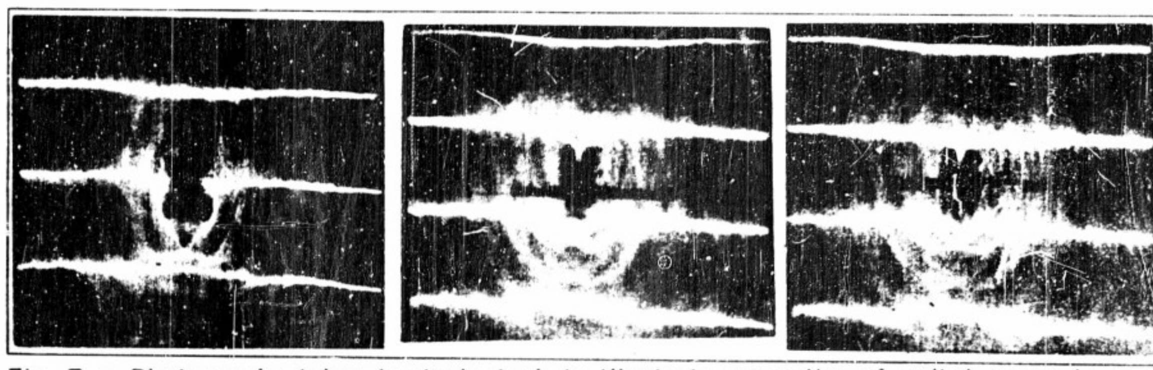

Fig. 7 - Photographs taken in ripple tank to illustrate generation of radial waves by a floating body in a regular train of waves

(Bell and Howell - Eymo) with a 50 mm. focal length lense and running at 48 frames per second. To reduce the error due to paralax, special care was taken in mounting the camera so that the center of the lens was exactly at the same elevation as the still-water surface. A string-grid (0.10 foot intervals) was mounted on the plate-glass window of the channel for a scale. For a time record an electrically operated clock, graduated in 1/100 seconds, was mounted in the field of view of the camera. The general set-up is shown in Figures 10, 23 and 24.

An independent record of the water surface-time history was obtained by mounting a double wire resistance element in front of and another one behind the floating body, the output being recorded on a Brush recorder. The comparison between the wave heights obtained from the movies and that of the Brush records was found to be good (± 5%).

Model Basin: The first experiments were made in the wave channel using all the described models. However, due to the narrowness of the channel, waves generated by the oscillating body were reflected from the walls (Figure 7*), and it was thought that they might affect the results.** Because of this problem later experiments were made in a large wave basin (approximately 65 by 65 feet). The water depth was limited to a maximum of 1.50 feet and resulted, for the longer wave lengths, in shallow-water waves instead of deep-water waves . The wave machine was of the flap type and extended across the total width of the basin. The wave period could be varied by changing the speed of the driving motor, and the wave amplitude could be changed by adjusting the throw on the crank arms connected to the wave flaps. On the opposite end of the basin the wave energy was absorbed, without reflection, on a sloping sandy beach. The general view of the wave basin is shown in Figure 5.

The greatest difficulty encountered in performing experiments in the large basin was the photographing of the motion of the body so that evaluation of the data would be relatively simple and the results reliable. There was no possibility of mounting the camera lens at the same elevation as the still-water level; consequently, the following procedure was used: First a reference line was established by stretching a piano-wire parallel to the still water level at a known distance above it. The camera then was mounted so as to obtain a desirable field of view, with the lense as close as possible to the water surface. A few feet of film was exposed to obtain a picture of the grid (see Figure 8a) which was placed in the same vertical plane as the reference line. During the actual test runs the grid was removed, but the camera and reference line were kept exactly in the same position. A sample of an actual run is given in Figure 8b. Enlargements of the movie frames were obtained and by using the grid as shown in Figure 8a, the scale and the reference line were transferred to transparent paper. The enlargement of the movie frames was kept constant, and all the data were evaluated by using a transparent scale as an overlay. The reference line, which was in both the scale and the photographs, was used as a guide line.

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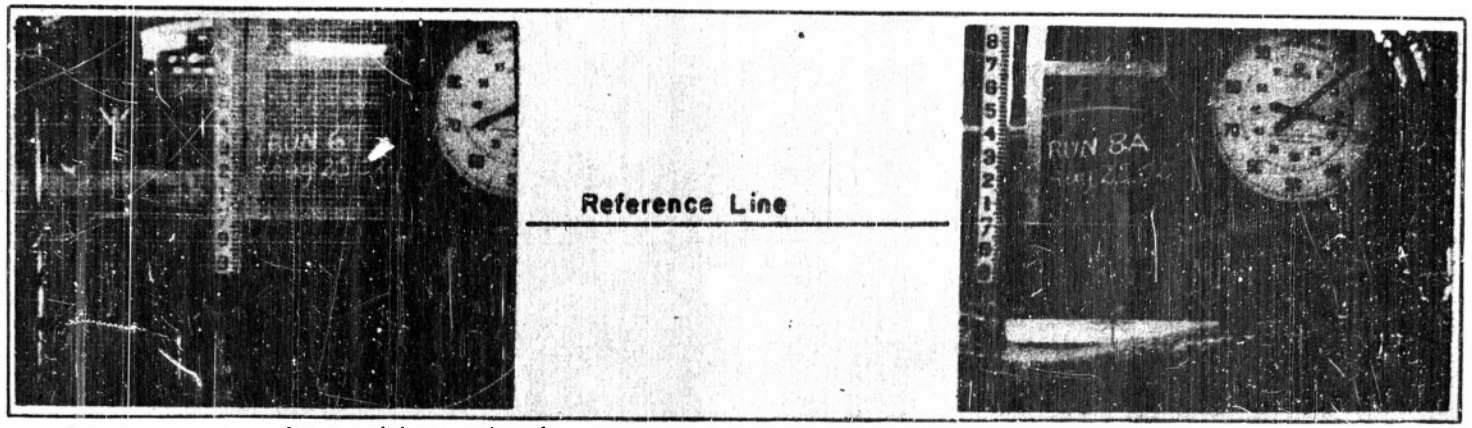
* Photographs of Figure 7 were obtained in a Ripple Tank using a small rectangular block about 3 inches long, floating in a regular train of waves. A 16 mm. movie camera was used to record the data (5).

** Later it was discovered, however, that the reflected waves were so small (as compared with the original wave motion) that they had hardly any influence on the results. This statement is true only when the ship is not underway.

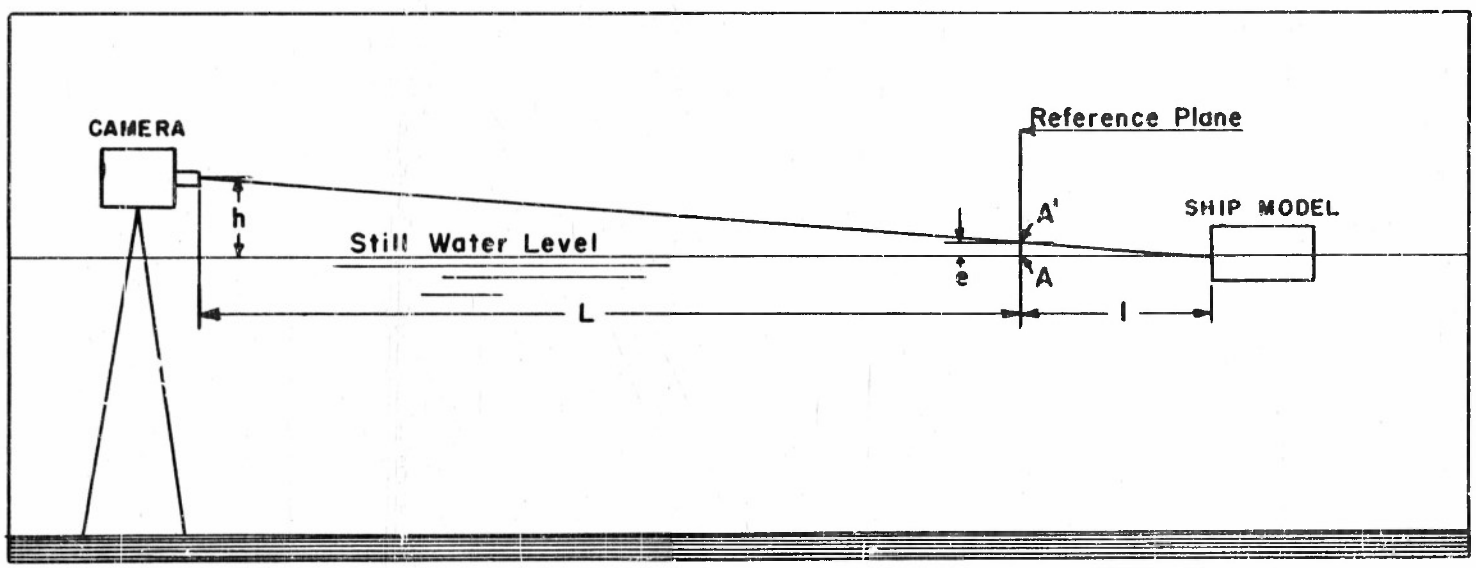


a. Photograph with grid to obtain a scale for evaluation

b. Photograph as taken of an actual run

ARRANGEMENT OF EXPERIMENT IN OPEN WAVE BASIN

FIG. 8



SKETCH OF SET-UP IN OPEN WAVE BASIN

FIG. 9

Special care was taken during the experiments to keep the front side of the model in exactly the same vertical plane as the reference line. When the model moved too much out of this reference plane, the run was discarded and repeated. The error involved when the model moved out of the line can be determined using the following simple computation, and the sketch shown in Figure 9. When the elevation of the camera lense above the still water is h and the distance of the camera from the reference plane is L, then the reading at the reference plane will be A' instead of A when the model moves out of the plane by the amount, l. The error involved would be e. By simple geometrical relationship,

$$\frac{h}{e} = \frac{L + l}{l} \qquad (1)$$

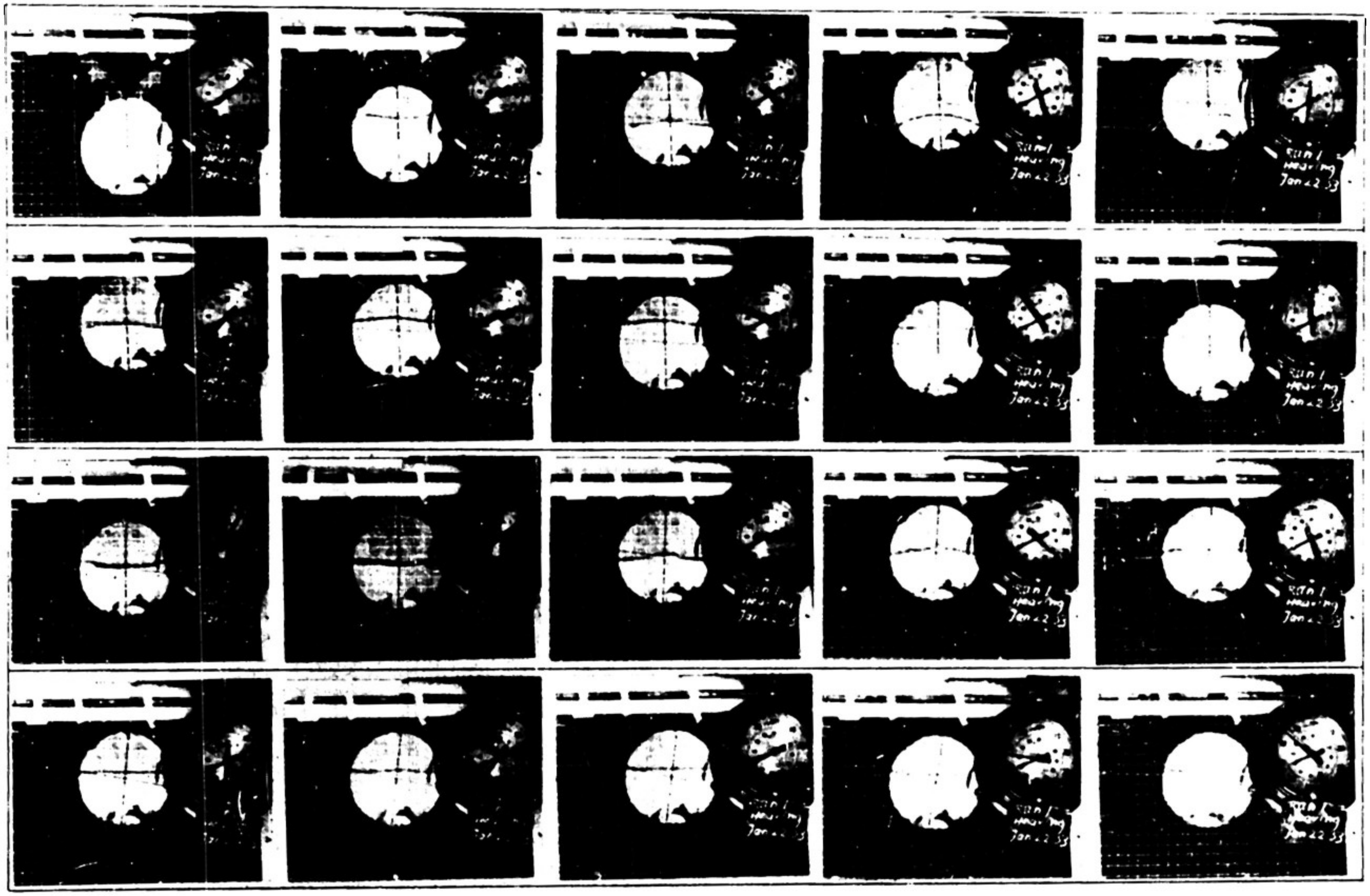
or considering that l is very small compared with L, we have $h/e = L/l$
If a maximum error $e = 0.005$ foot is allowed with the distance L being equal to 20 feet and $h = 0.5$ foot, then a movement of l of only 0.2 foot is allowable. This means that by allowing an error of 0.005 foot, the front side of the model may move away from the reference plane by an amount of 0.2 foot on either side. When the model moved away from the reference plane by a greater amount the run was discarded. In general the model kept itself very well in line. More accuracy could have been obtained by moving the camera closer to the water surface and increasing the distance from the camera to the reference plane.

III. EXPERIMENTAL WORK

Preliminary tests were made in still water to determine the damping coefficients for heaving and pitching for the models. After these had been determined, tests were performed in different wave conditions. During these tests, the following conditions were maintained: (1) all waves were long-crested; (2) the model was not underway, but was freely floating; (3) the longitudinal axis of the model was parallel to the direction of wave travel.

Damping: The first runs made were to determine the damping coefficients for heaving and pitching motions. For the heaving motion the model was pressed down in still water, using a pointed stick in the center of the model, and then suddenly raising the stick. For the pitching motion one end of the model was pressed down in still water and then released. The motion was recorded on the movies and later analyzed for motion time histories. A sample of the experimental work is shown in the series of photographs of Figure 10, the pictures being enlargements of the 35 mm. movie film. The grid lines in the photographs were 0.1 foot apart. For the circular cylinder the initial displacement was obtained by pressing the cylinder down in still water by hand as can be seen in the first picture of Figure 10.

Motion in Non-Uniform Waves: For the runs in the wave channel, the models were placed approximately 20 feet from the wave generator in still water, with longitudinal axis parallel to the channel walls. The flapper of the wave generator was disconnected from the driving motor and operated manually, changing the period of amplitude of the motion continuously to obtain a train of irregular waves. The motions of the models were recorded on 35 mm. film. In the large wave basin the manual operation of the wave generator was not possible. Consequently, to obtain an irregular train of waves, the wave generator was started and then stopped after a short period of time. The generator started slowly and then accelerated to a constant period. When stopped, the generator decelerated slowly to a stop. The result was that the wave train started with long waves which decreased in length as the operation continued



EXPERIMENT TO DETERMINE THE DAMPING COEFFICIENT FOR THE HEAVING MOTION OF A CYLINDER - Time between pictures approximately 1/10 of a second, as can be seen from clock; strings on grid are 0.1 foot apart.

FIGURE 10

until a constant wave length was reached. After stopping the generator, the wave length increased steadily until the generator stopped completely. Due to the different velocities of travel for the different waves, the train was fairly irregular when it reached the model, located approximately 35 feet from the generator.

Motion in Uniform Waves: In addition to the non-uniform waves, the experiments were also made for a regular train of waves (constant period and height). In previous experiments with uniform waves it was found that there was no significant difference in results obtained in the 1 foot wave channel and those obtained in the wave basin.* It was decided, therefore, to complete the experiments with uniform waves in the wave channel.

After the desired wave period and steepness were obtained, the model was placed in still water and the wave generator started. The motion was recorded only after a steady state was reached.

IV. RESULTS

In this report only a sample of the results will be given. Most of the data are presented in the report by Fuchs and MacCamy(2) and compared with their theory on the motion of a ship in non-uniform long-crested waves. Comparisons of the Fuchs-MacCamy theory with experimental data were good. In this report a comparison of the Weinblum theory for regular sine-waves is made. Weinblum's theory (7) is based partly on the work of Kryloff(8) and seems to be the most popular to date.

Evaluation of Data: The evaluation of the data on the movie film was very tedious as each frame had to be viewed for the shorter wave lengths; for the longer waves every third frame was sufficient. From each frame the following data were obtained: (1) the elevation of the reference point on the model, (2) the elevation of the water surface at the center line of the ship, (3) the pitching angle Θ and (4) the horizontal location of the reference point on the model.

A reference point was located on the side of the model in such a manner that it never was below the water surface. The readings for the location of reference point and the pitch angles were comparitively easy to obtain. More care was required to obtain the water-surface elevation. The readings of the elevation of the model and the water surface in the wave channel were taken from the side of the model and not from the window of the channel, in order to reduce errors. To define the water surface on the side of the model a very clearly distinguishable vertical line was drawn through the reference point (see Figure 2) at the center of the model. The water surface then was defined in the pictures through a break in the straight line, the break resulting from the different angles of refraction of light in air and water.

The accuracy of measurements were $\pm$ 0.002 foot for lengths, and $\pm$ 0.2 degree for pitch angles. The data were plotted as elevation-time and angle-time histories.

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* This statement is true only for the case when the model is not underway, and is not true for the determination of experimental damping coefficient as described above. Damping is in first line due to the generation of waves and in a narrow channel these generated waves will be reflected by the channel walls and so effect the results.

<u>Damping:</u> For oscillations in still water the equation of motion for heaving is

$$M\zeta'' + N_1 \zeta' + \rho A g \zeta = 0 \tag{2}$$

and for pitching is

$$I\,\Theta'' + N_2\,\Theta' + W m\,\Theta = 0 \tag{3}$$

where:

ζ = upward displacement of the center of gravity from the still-water level

M = effective mass of the body for vertical oscillations

A = water plane area

ρ = density of water

N_1, N_2 = damping coefficients to be determined experimentally for heaving and pitching motions

I = effective moment of inertia of the body for pitching oscillations

W = ρV = displacement of the body in equilibrium

m = longitudinal metacentric height

Θ = pitching angle

V = displaced volume of water

T_1, T_2 = natural periods for heaving and pitching motions.

Replacing N_1/M by $2b$ and rewriting the equation for the heaving motion,

we have
$$\zeta'' + 2b\zeta' + \omega^2 \zeta = 0 \tag{4}$$

where $\omega^2 = \dfrac{\rho A g}{M}$

The solution of this equation is known to be

$$\zeta = e^{-bt}(A_1 \cos \sigma t + B_1 \sin \sigma t) \tag{5}$$

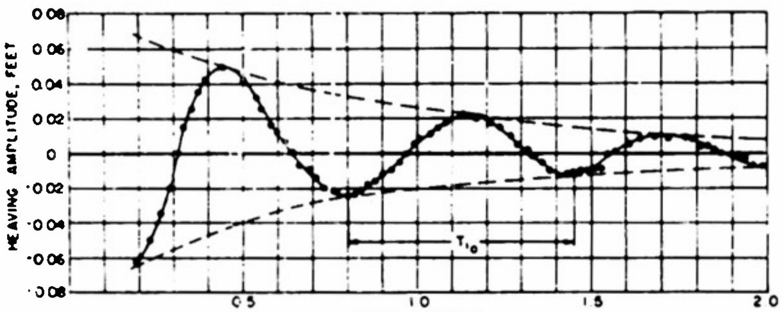
Here A_1 and B_1 are constants and $\sigma = \sqrt{\omega^2 - b^2} = 2\pi/T$, where T_1 is the natural period of the heaving motion. The envelope to this curve has the form $y = e^{-bt}$ const. Taking the logarithm of both sides, we have $\log y = -bt + \text{const}$. This means that we can obtain b as the slope of the envelope to the experimental damping curve when replotted on semi-log paper. The effective mass M can be computed from the following relationship:

$$\sigma = \sqrt{\omega^2 - b^2} = \frac{2\pi}{T_1} = \sqrt{\frac{\rho g A}{M} - b^2}$$

T_1 and b are experimentally determined, hence we have

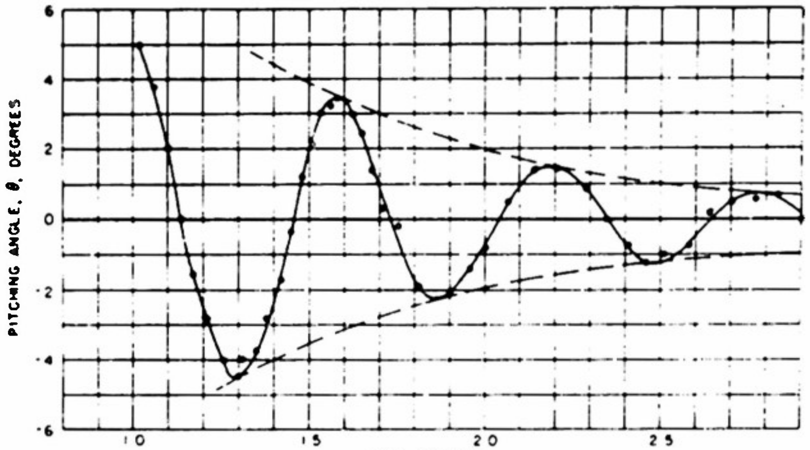
$$M = \frac{\rho g A}{(2\pi/T_1)^2 + b^2} \tag{6}$$

and
$$N_1 = 2Mb \tag{7}$$



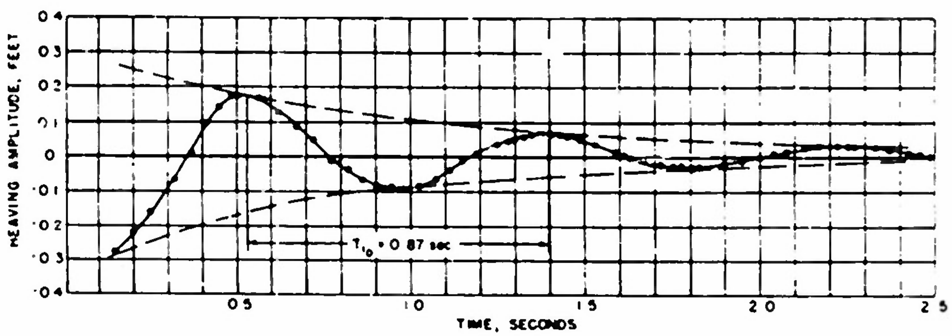
DAMPING CURVE FOR HEAVING MOTION OF RECTANGULAR BLOCK IN STILL WATER

FIG 11-a



DAMPING CURVE FOR PITCHING MOTION OF RECTANGULAR BLOCK IN STILL WATER

FIG 11-b



DAMPING CURVE FOR HEAVING MOTION OF CYLINDER IN STILL WATER

FIG 12

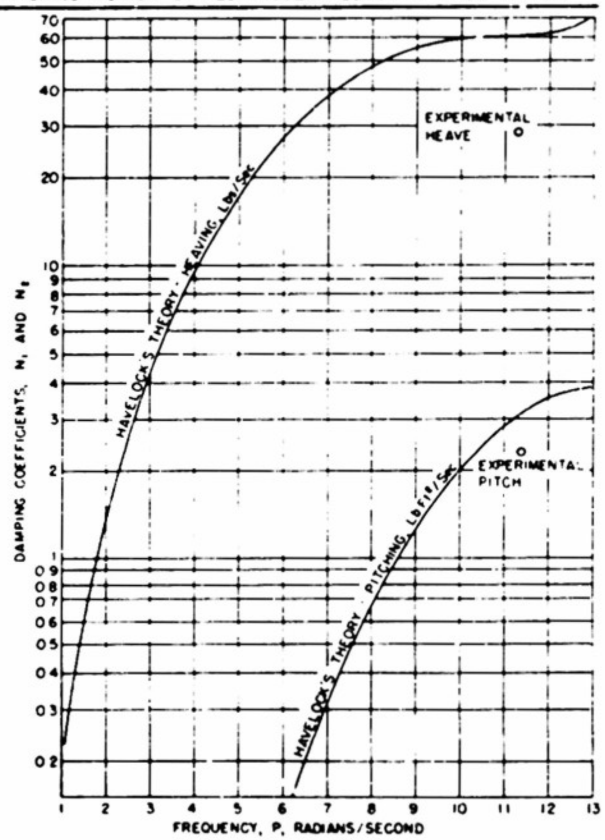


FIG. 13

HEAVING AND PITCHING DAMPING COEFFICIENTS
AS COMPUTED FOR RECTANGULAR BLOCK

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The damping coefficient N_2 can be obtained similarly by solving the differential equation for the pitching motion.

For the rectangular block it was found that $N_1 = 29$ lbs/sec and $N_2 = 2.28 \frac{\text{lbs.ft}^2}{\text{sec}}$. These results are compared in Figure 13 with the theory of Havelock.(6) Both the experimental results are below the theoretical curve, with N_2 showing better agreement.

Heading and Pitching: Samples of the experimental results of the model in non-uniform waves are given in Figures 14, 16, 19 and 20. The runs are illustrated by photographs in Figures 17 and 18. Figure 17 represents a short section of experiment with relatively short wave lengths as compared with the length of the model, while in Figure 18 the wave length is slightly longer and the waves less steep than in Figure 17.

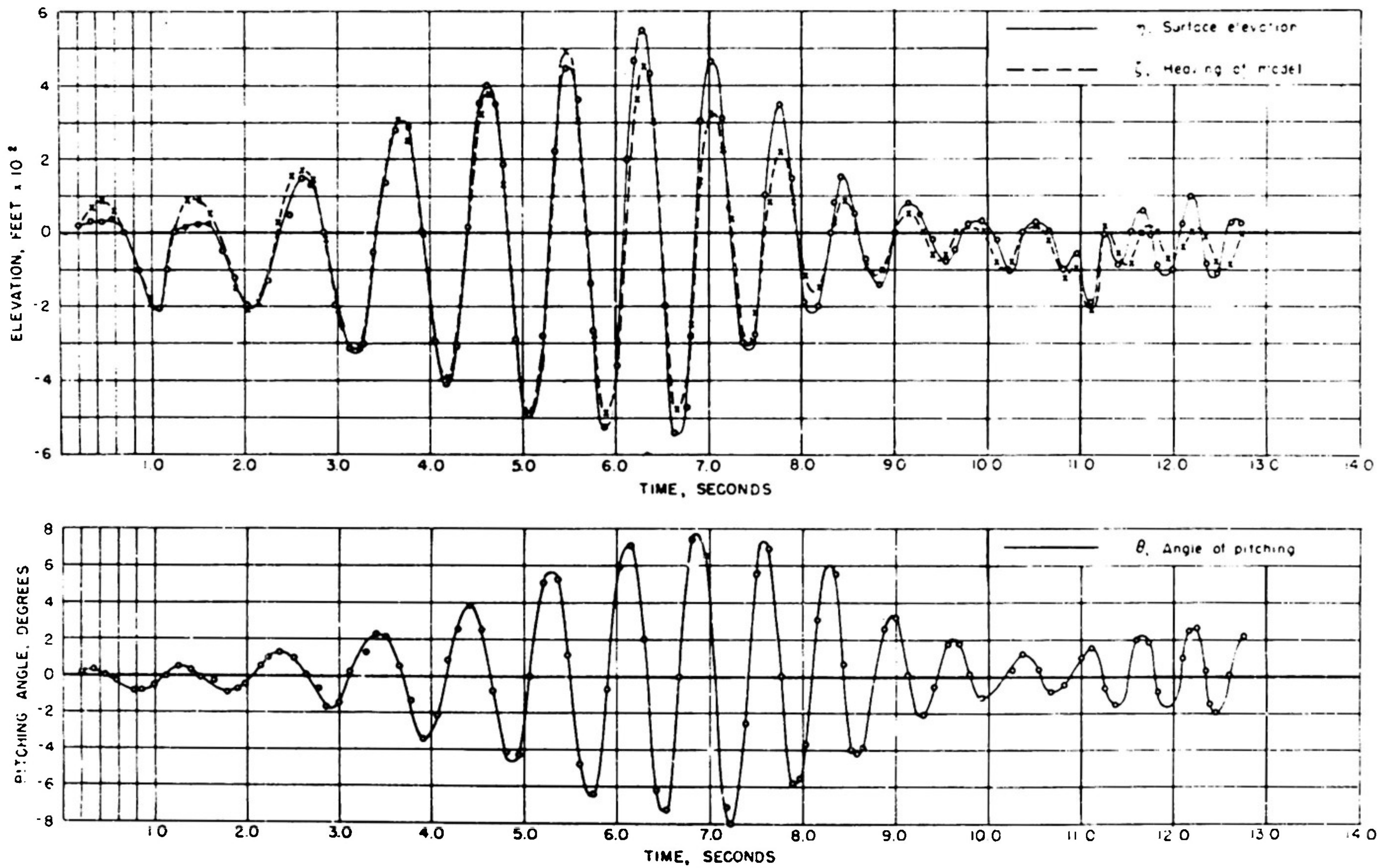
The results in uniform train of waves, as obtained in wave channel, are given in Figure 21 for the rectangular block and in Figure 22 for the ship model. The runs are illustrated by photographs in Figure 23 and 24.

Non-Uniform Waves: In Figures 14, 16, 19 and 20 the heaving amplitude of the model is compared with the wave amplitude, and the pitching angles are given as the angle-time history. In the case of long waves the model follows almost exactly the wave motion--that is, the amplitude of the heaving motion is very nearly equal to the amplitude of the wave motion, and in some cases might even exceed the latter. As will be seen later, this condition is in accordance with the theory. As the wave length decreases with respect to the length of the model, the heaving motion of the model decreases, and may even become very small compared to the wave motion. This condition occurs at the tail-end of a non-uniform train of waves with decreasing wave periods. Visually it can be seen in Figures 17 and 18. In Figure 17, where the wave lengths are comparatively short, the waves move along the side of the model with no appreciable heaving of the model; that is, the water surface elevation, measured on the side of the model, is changing continuously and is related to the wave motion. In Figure 18, however, the wave length is longer than that occurring in Figure 17 and it can be seen that the model follows the waves smoothly and the water-surface elevation, measured on the side of the model (at the center line), remains almost constant.

Uniform Waves: Figures 21 and 22 demonstrate the ship motion in a uniform train of waves. Figure 21 represents an experiment with the rectangular block and Figure 22 that with the ship model. (The experiments with uniform waves were conducted in the wave channel) In Figures 21 and 22 one can see clearly how the amplitude of the heaving motion decreases as the wave period decreases.

In Figure 21a there is almost no heaving motion present for the period $T = 0.48$ sec. and with the wave length almost equal to the model length. For the short wave periods, a shift in phase also can be seen. This indicates that the maximum heaving elevation will not occur at the same time that the wave crest passes the center of the ship, but rather, a short time later. The shift in phase seems to be considerable when the wave period is equal to the natural period of oscillation. The shift in phase can be followed in Figures 21b and 22a. In Figure 22a the shift is equal to approximately 180°. This means that the heaving of the ship reaches its maximum value at the moment when the wave trough passes the center of the ship. This occurrence should be seriously considered for it might result in considerable stresses in a ship's structure even though the heaving amplitudes are very small.

Figures 23 and 24 are illustrations of the experiments with a uniform

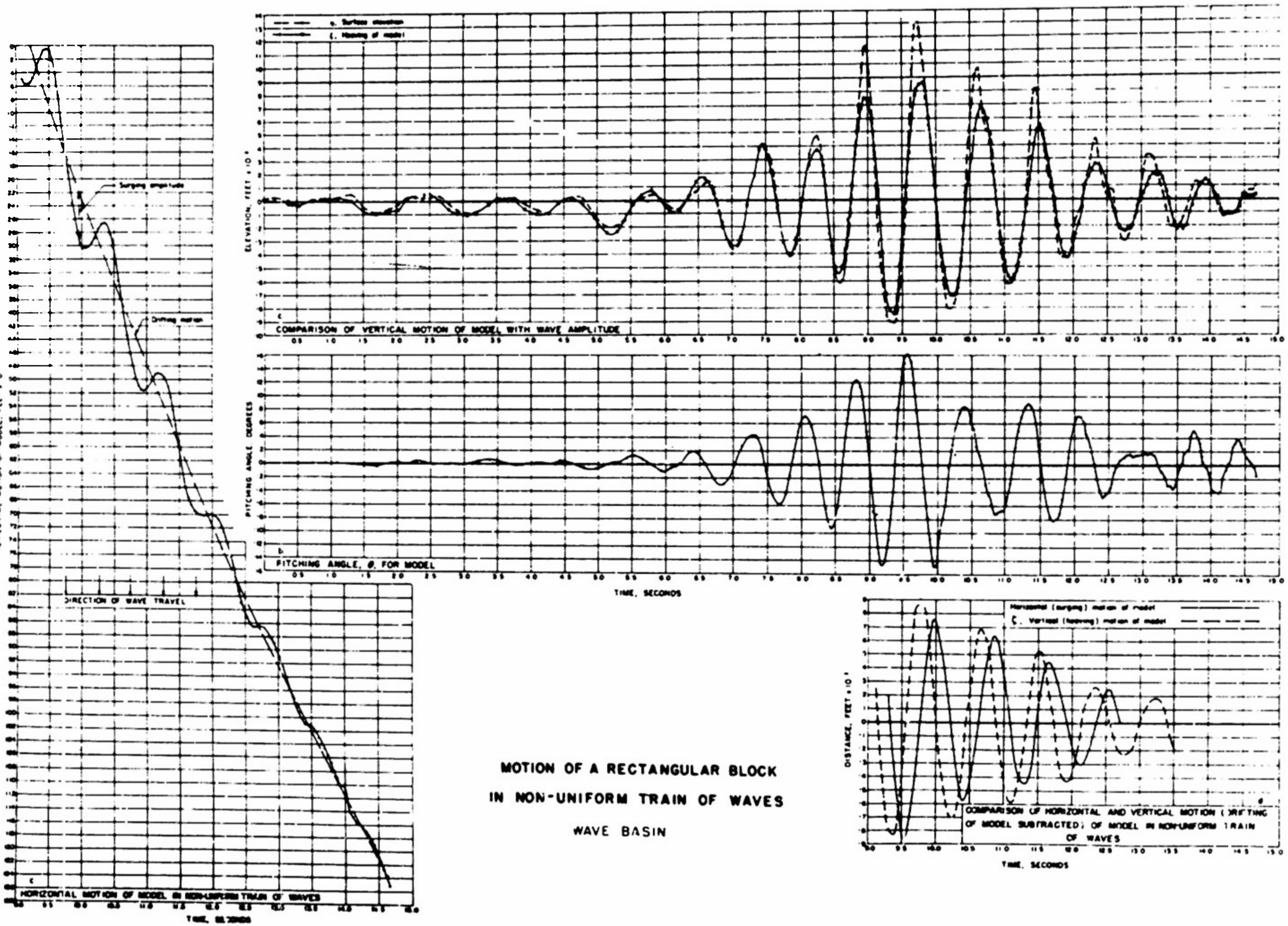


OSCILLATION OF FLOATING RECTANGULAR BLOCK IN GRAVITY WATER WAVES

NON-UNIFORM WAVES IN WAVE CHANNEL

HYD 6594

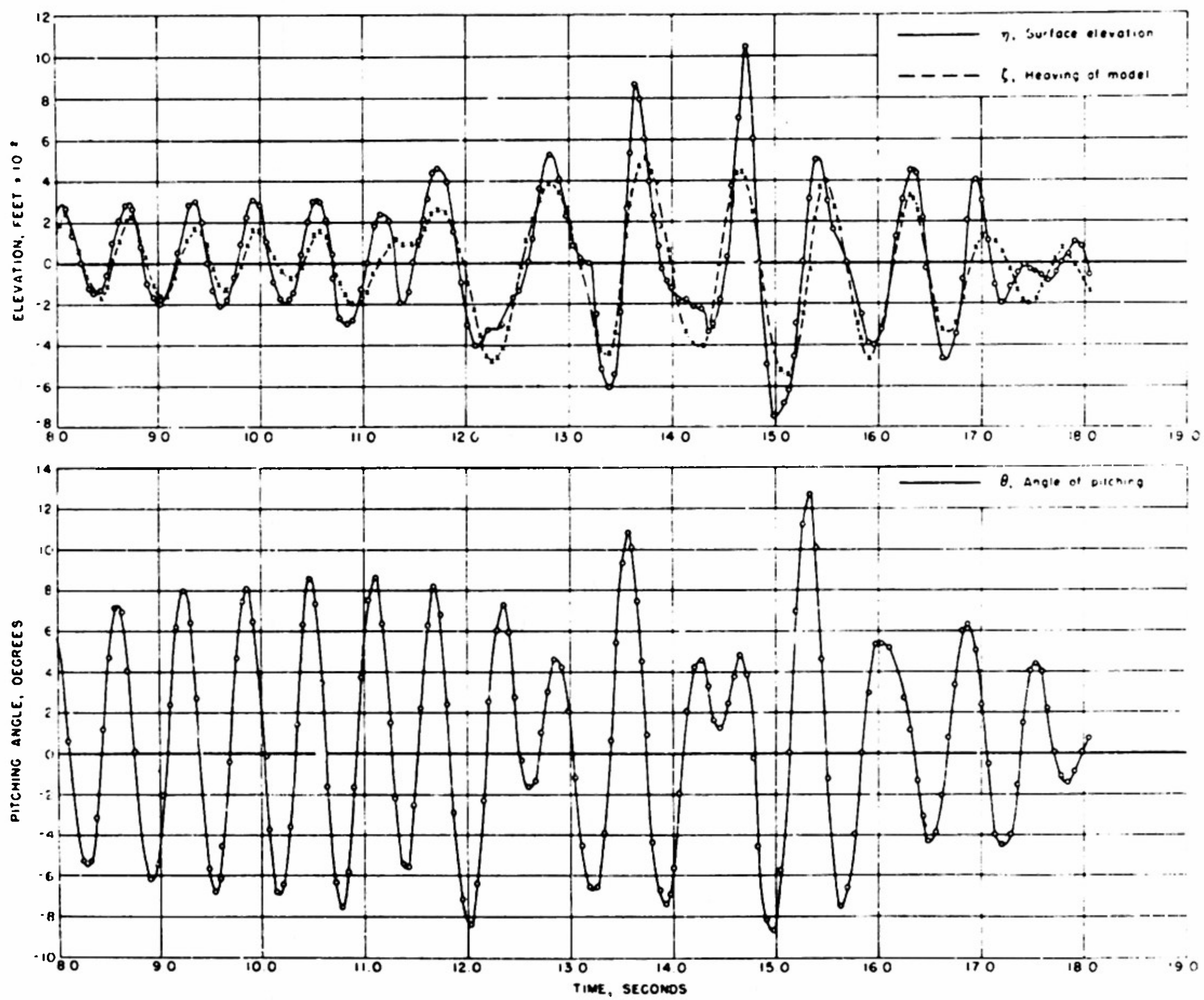
FIGURE 14



MOTION OF A RECTANGULAR BLOCK
IN NON-UNIFORM TRAIN OF WAVES

WAVE BASIN

FIGURE 15



OSCILLATION OF FLOATING RECTANGULAR BLOCK IN NON-UNIFORM TRAIN OF WAVES

WAVE BASIN

FIGURE 16

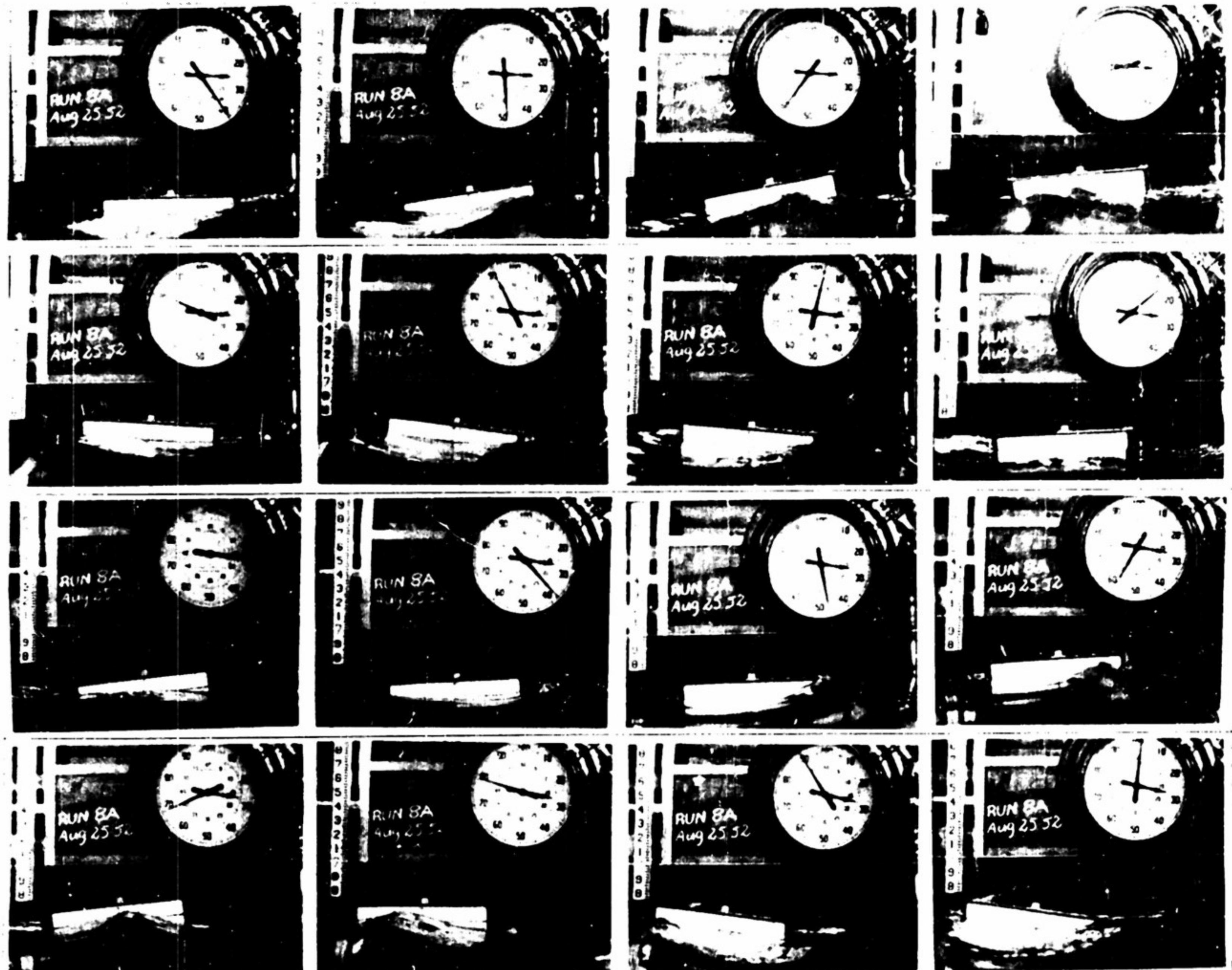


ILLUSTRATION OF RUN 8A - Rectangular block in a train of non-uniform long-crested waves; time interval between pictures approximately 0.1 second.

FIGURE 17

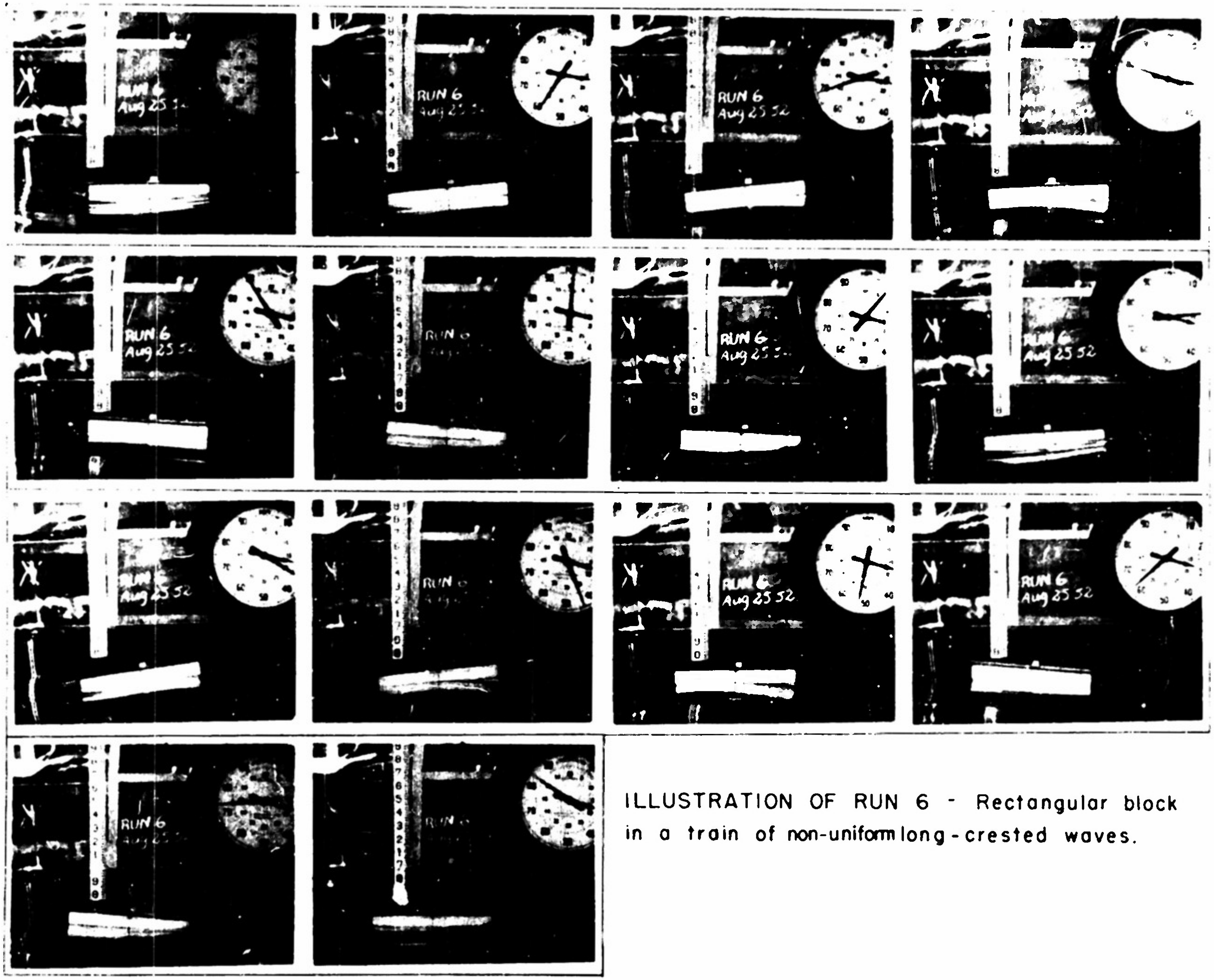
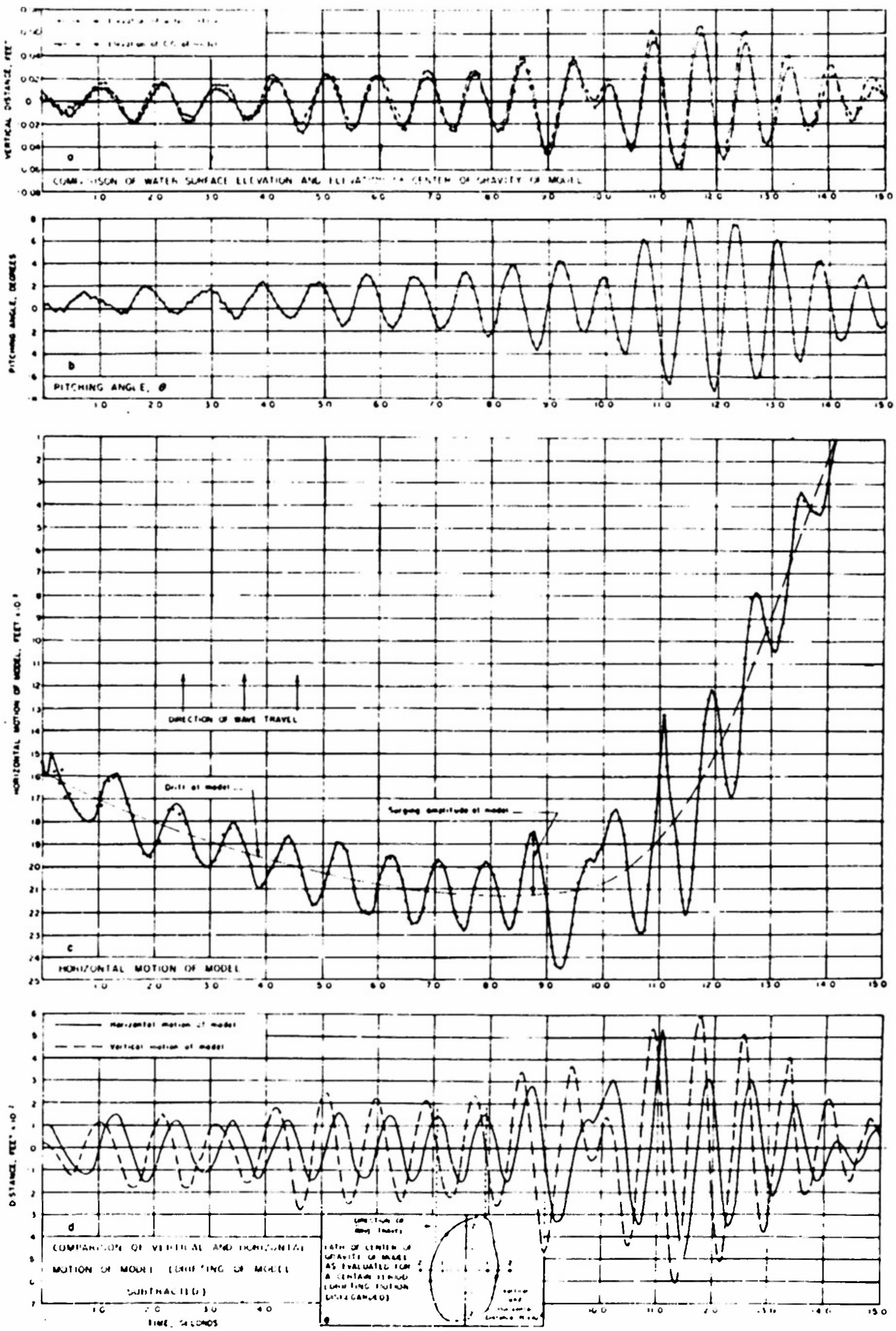
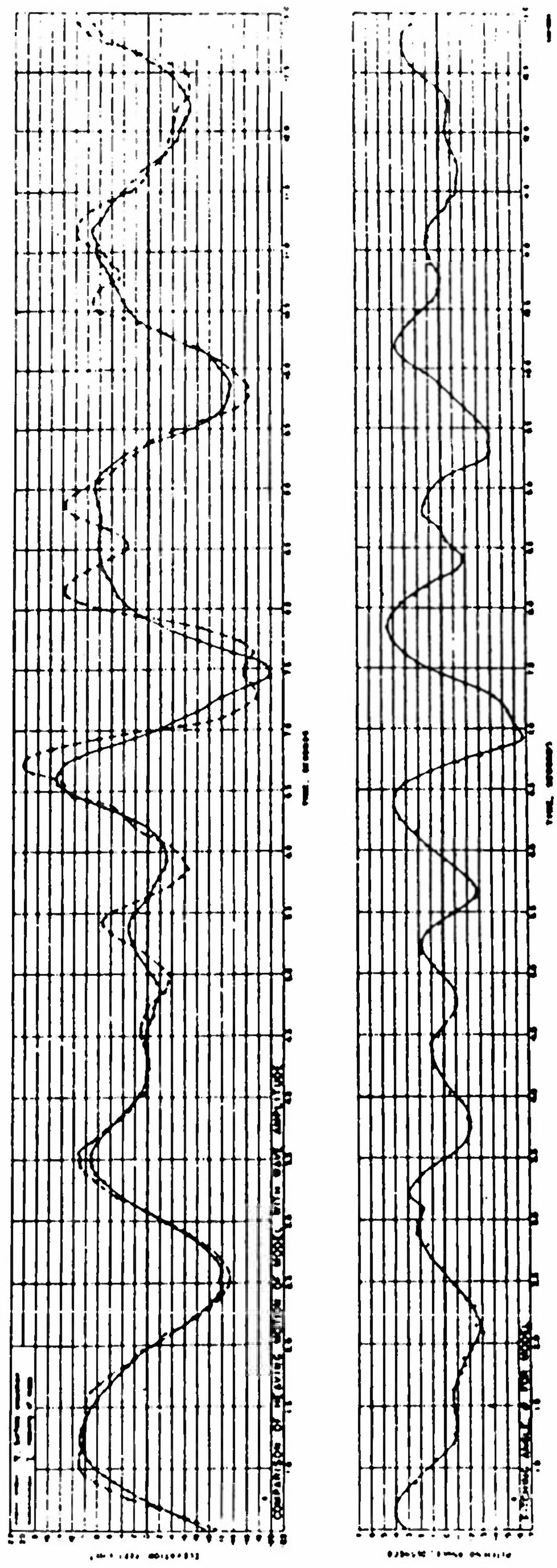


ILLUSTRATION OF RUN 6 - Rectangular block in a train of non-uniform long-crested waves.

FIGURE 18



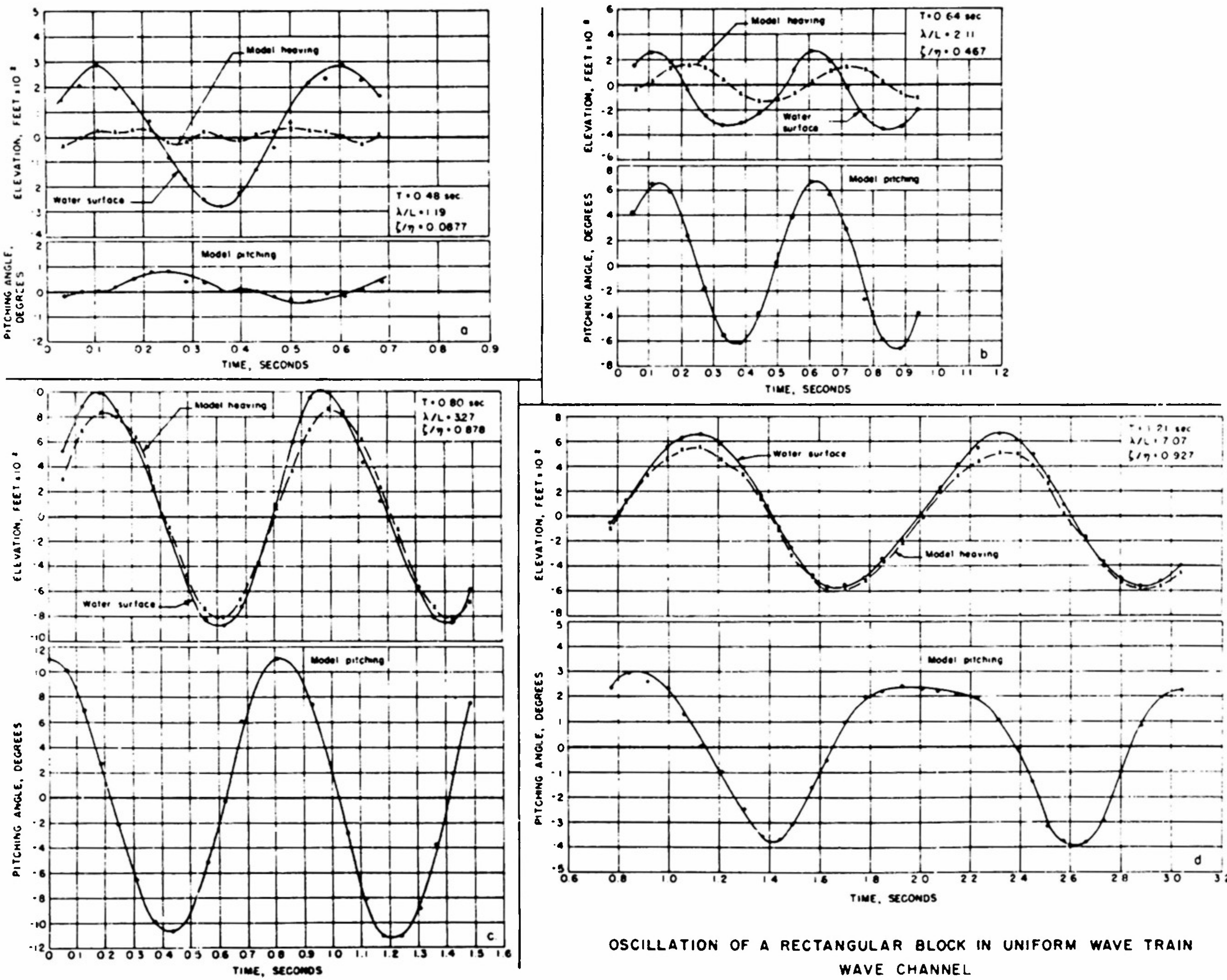
MOTION OF A FLOATING RECTANGULAR BLOCK IN NON-UNIFORM TRAIN OF WAVES
WAVE BASIN



OSCILLATION OF A SHIP MODEL IN NON-UNIFORM TRAIN OF WAVES

WAVE CHANNEL

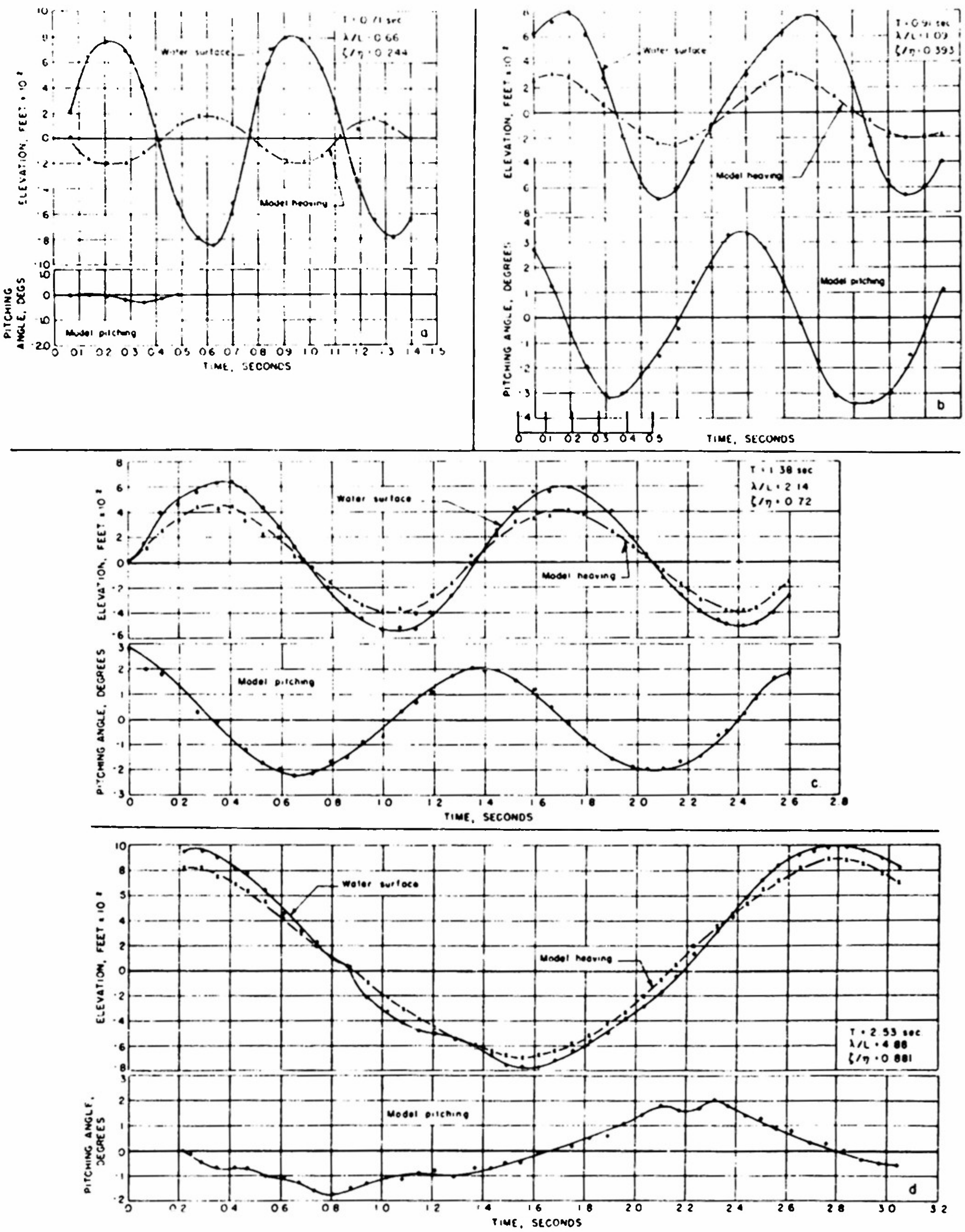
Figure 20



OSCILLATION OF A RECTANGULAR BLOCK IN UNIFORM WAVE TRAIN
WAVE CHANNEL

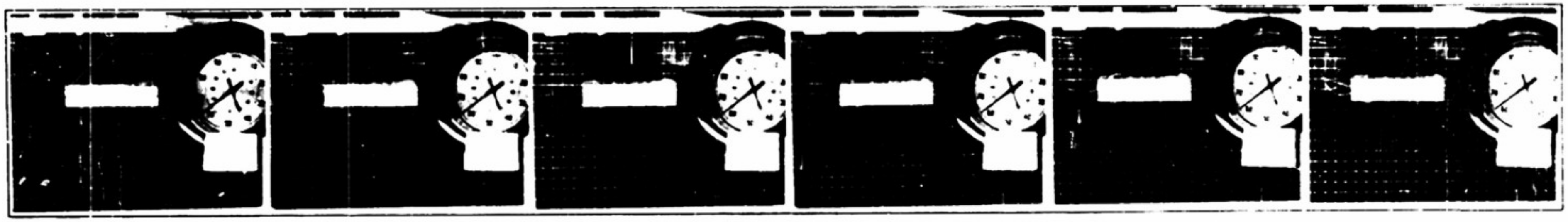
H-D-6599

FIGURE 21

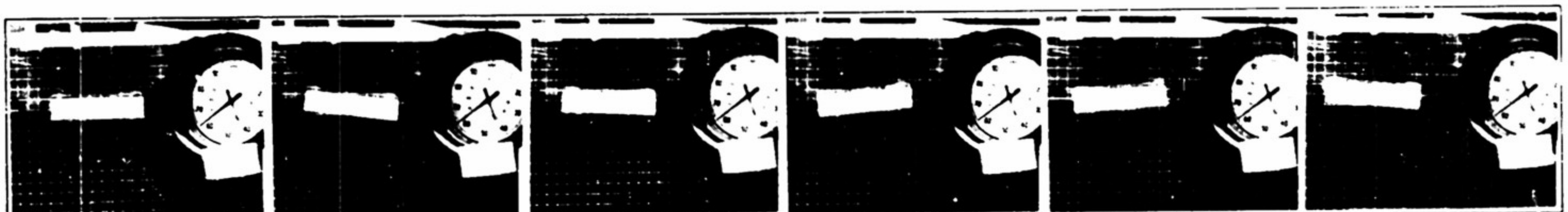


OSCILLATION OF A SHIP MODEL IN UNIFORM WAVE TRAIN
WAVE CHANNEL

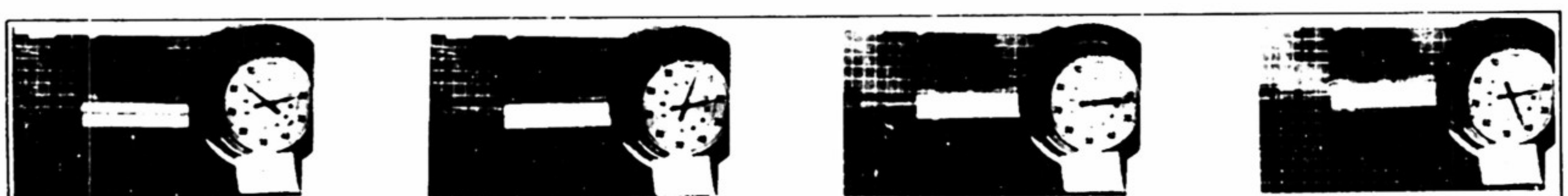
FIGURE 22



a. T = 0.48 second; λ = 1.19 foot; λ/L = 1.19



b T = 0.64 second; λ = 2.1 feet; λ/L = 2.1



c

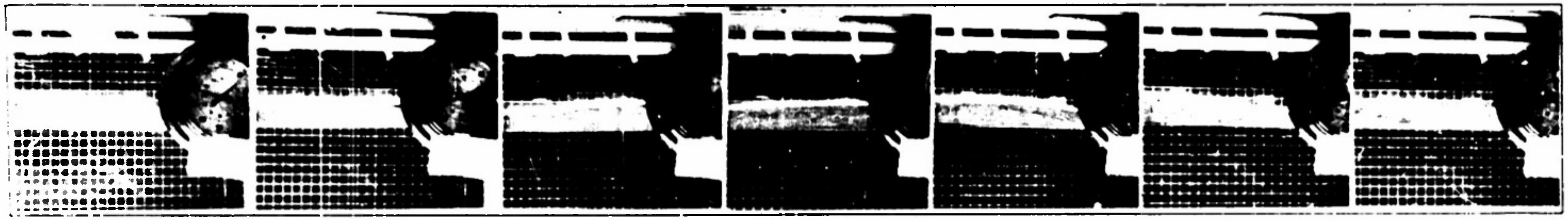
T = 1.08 second

λ = 5.9 feet

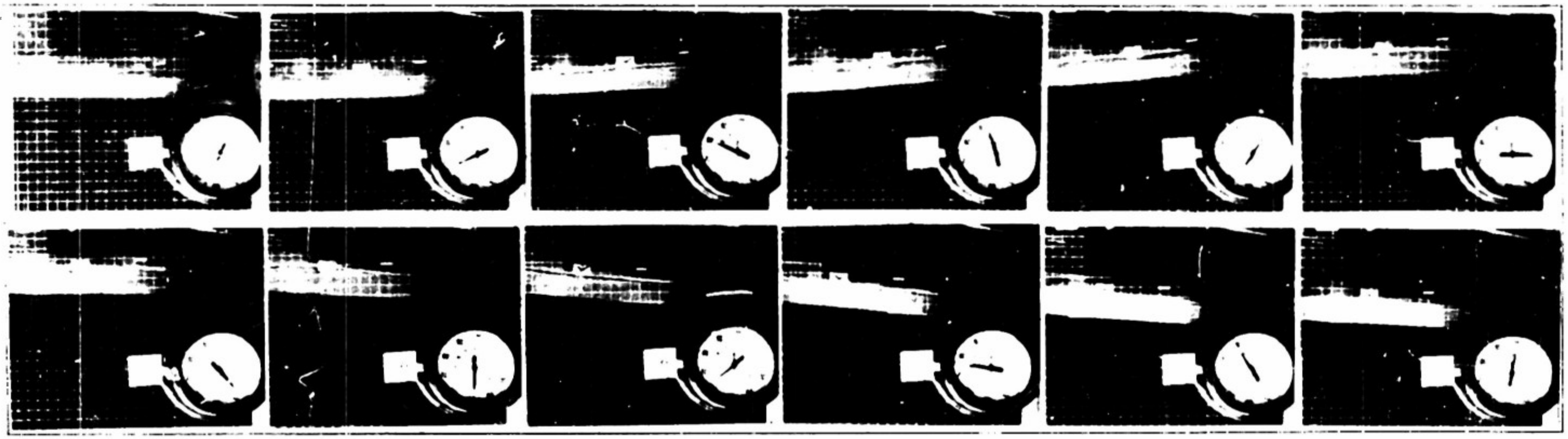
λ/L = 5.9

Rectangular Block in uniform train of waves, in wave channel

FIGURE 23



a T = 0.8 second, λ = 3.30 feet, λ/L = 0.845



b T = 1.5 second, λ = 9.9 feet, λ/L = 2.54

Ship Model in uniform train of waves, in wave channel

FIGURE 24

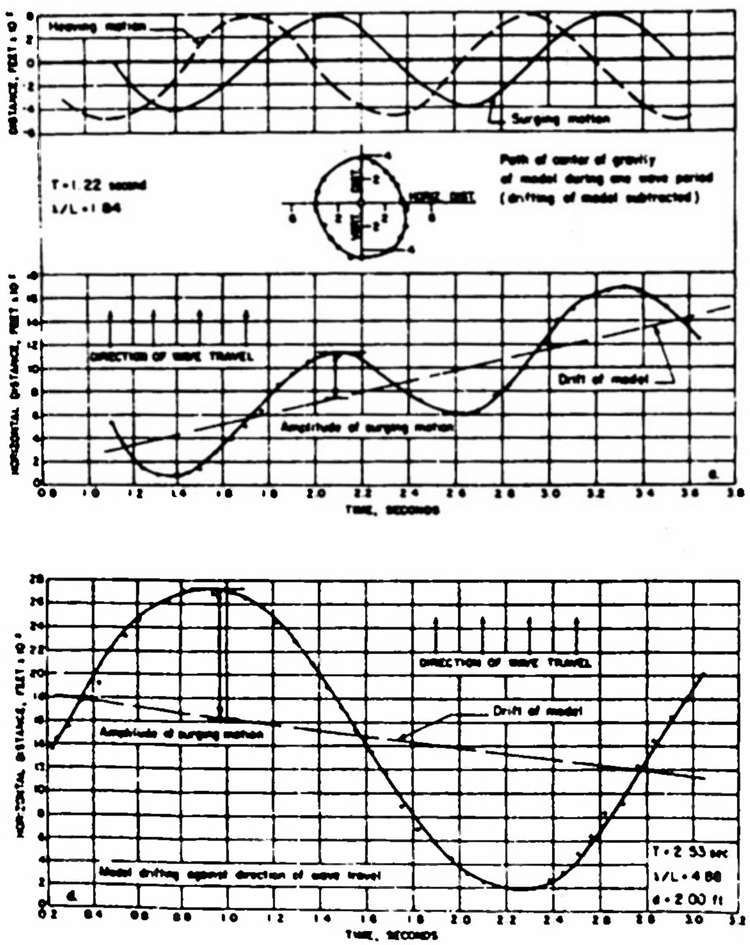
train of waves. Figure 23 illustrates the rectangular block and Figure 24 illustrates the ship model tests. Figures 23a and 24a demonstrate the case when the wave length is very short compared to the length of the model. It is of interest to note how the waves pass the model without causing the model to heave or pitch. The only appreciable motion of the ship caused by the waves is the drift of the model in the direction of the wave travel. Figure 23b demonstrates the shift in phase. The model reaches its maximum elevation and the maximum pitch angle, after the wave crest has passed the center of the model. Figures 23c and 24b demonstrate the case of long waves; here the model follows the water surface smoothly.

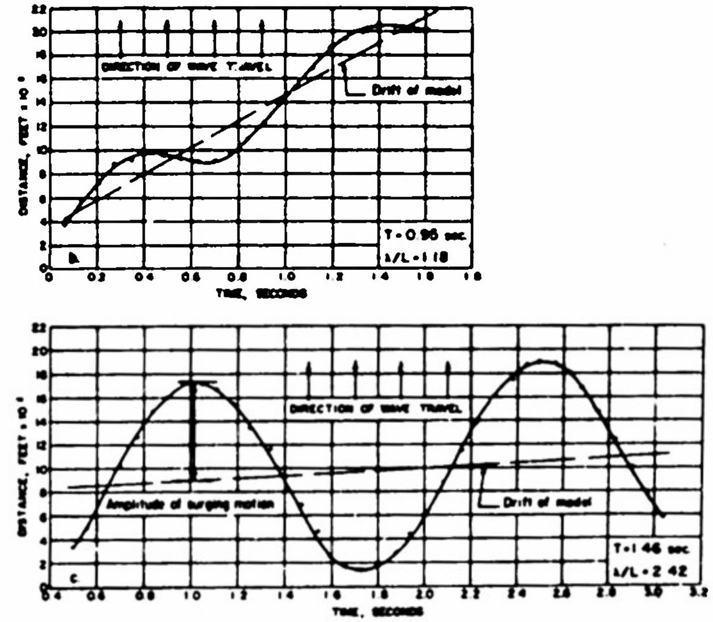
Slamming: It appears that the phenomenon of slamming or pounding seems to be related to a shift in phase. Slamming occurs when the amplitude of heaving or pitching becomes so large, relative to the wave, that a portion of the bottom of the hull emerges, to re-enter the water with an impact. This may easily reach such an order of magnitude that the periodicity of motion is destroyed. The forces depend essentially upon the relative vertical velocity between the bottom of the ship and the water surface. When we have, as an example, a 180° shift in phase, the relative velocity between the ship bottom and the water surface is naturally very high, for the motion occurs in opposite directions. The shift in phase of the heaving motion is always connected with a shift in phase of the pitching motion. This means the slope of the wave and pitching angle of the ship may have opposite directions which again might cause the forebody of the ship to emerge and to re-enter the water with heavy impact.

It was found that slamming occurs for both uniform and non-uniform waves when a certain condition was reached. However, slamming was prevented in the experiments as it was found to destroy the periodicity of the motion, as the data could not be used to compare experimental results with theory. In Figure 23b ($T = 0.64$ sec.) the slamming condition has almost been reached. In the third picture of the illustration the front-end of the model is almost submerged while in the fifth picture it almost emerges. In this picture the shift in phase of heaving and pitching motions can clearly be seen as mentioned before.

Surging and Drifting Motion: In addition to the vertical motion of the model, the horizontal motion was measured and evaluated. A part of the reduced data is given in Figures 15c and d and Figures 19c,d and e for the non-uniform waves, and in Figure 25 for the uniform train of waves. In Figure 19, the horizontal motion of the model is shown for a train of non-uniform waves. As can be seen in Figure 19c, the horizontal motion consists of two parts: (a) the drifting motion of the model, and (b) the surging motion. In Figure 19c the total horizontal motion is plotted as the time-distance curve.

Surging: The surging and drifting motions of the model can be separated by drawing a center-line through the experimental curve as shown in Figures 19c and 25 . This center line represents the drifting motion of the model. Using this center-line as zero-line, we can obtain the amplitudes of surging motions. Thus, Figure 19d is obtained by replotting 19c by using the center-line as zero-line . In the same plot the surging motion is compared with the heaving motion. In Figure 19e the path of center of gravity of the model is evaluated for a certain period without considering drift velocities. As to the comparison of the heaving and surging motion, it seems that they are almost equal for the longer wave lengths - this fact indicates that the center of gravity of the model moves along an almost circular path. When the wave period is decreasing, the circular path seems to become elliptic with the longer axis in vertical direction (compare Figure 19e and 25a). The surging force is





HORIZONTAL MOTION OF A SHIP MODEL IN UNIFORM WAVE TRAIN

FIG. 25

DRIFTING VELOCITY OF MODEL
AS A FUNCTION OF
RELATIVE WAVE LENGTH λ/L

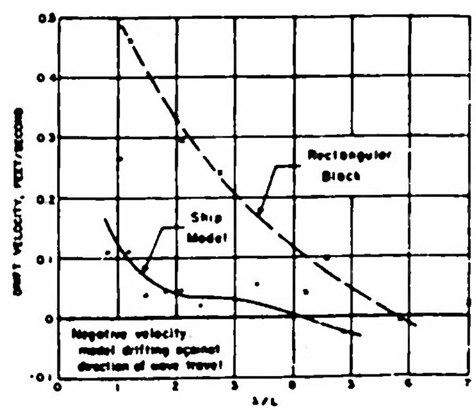


FIG. 26

the horizontal component of wave force along the ships longitudinal axis and depends thus on λ/L in a similar manner as the heaving force does. The magnitude of the surging motion is much higher than is generally thought (see Figure 19d,e and 25a). The immediate conclusion of this is that the constant velocity towing tests in a seaway may give doubtful resistance values. Much better results might be obtained with self-propelled models under constant thrust conditions.

Drifting: Considering the drifting motion of a ship due to waves, when the ship is not underway and when it is not affected by winds or currents, one may ask the question whether the ship stands still or will it be moved by the waves in the direction of the wave travel, or will it move against the direction of the waves.

Gerstner's theory (1802) indicates that the water particles move in circles, (with fixed centers) and therefore no mass transport of water is effected by the waves. Stokes showed (1847) that there was a slow mass transport in the direction of the wave advance. Stokes' theory has been confirmed by experiments. According to this theory for particle motion, a ship floating without any velocity of its own might be expected to drift very slowly in the direction of wave travel, but never against the waves, when not affected by wind or currents. But looking at Figure 19c it can be seen that the ship model drifted against the waves at the beginning of a non-uniform train of waves, where the wave lengths were large. When the wave lengths were decreasing the drift gradually stopped and then reversed direction, and started to drift in the direction of wave advance. The model drifted with increasing velocity as the wave lengths decreased. This was found to be true for all the experiments. It seems to be obvious that the slight mass transportation of water particles could never give rise to the forcible drift of a model floating on such waves. Thus the drift of a ship must necessarily be attributed to the mutual action between the waves and the ship heaving and pitching in these waves. The drifting of a ship has been investigated by Kyoji Suyehiro(19) and his experimental results compared with a theory by Yoshihiro Watanabe(20). The drifting is explained by Watanabe by the hydrostatic buoyancy in the waves. Taking an elementary volume within the waves, a buoyancy can be calculated which acts on this volume (similar to Kryloff). If this buoyancy be resolved into horizontal and vertical components, and if the horizontal components be integrated over the total volume of the ship below the water surface, the horizontal force may be obtained.

The question has been investigated also by Havelock(18) who concludes that a satisfactory theory would have to include, among other factors, the effect of reflection of the waves by the surface of the ship. In his paper, however, this has been neglected in order to make tentative calculations from another point of view which associates the effect directly with the oscillation of the ship. The steady average drifting force was obtained depending upon the phase differences between the heaving and pitching motion and the periodic forces and couples due to the wave motion. The theory was compared with experimental results, and although available data were not suitable for detailed comparison, it was found that the calculations gave drifting forces of the right order of magnitude.

None of the available investigations indicate a drift in the direction

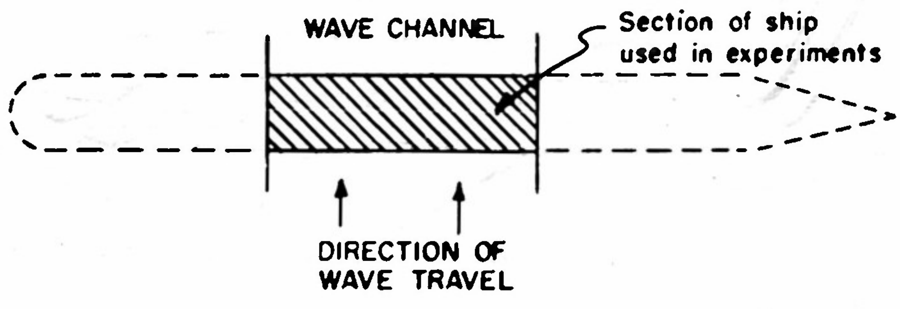
opposite to that of the wave advance; however, this might be due to the fact that all of the investigations are completed for short wave lengths, while the drifting against the waves has been found to occur at a wave length of about 5 to 6 ship lengths, depending upon the shape of the ship.

To obtain more data on this phenomena, additional experiments were made with uniform waves in the wave channel, using both the rectangular block and the ship model. Some examples are given for the ship model in Figure 25. The drift velocity was obtained from the slope of the center-line drawn to the experimental curve of horizontal motion as explained before. For uniform waves the center-line was always found to be a straight line, which indicates a constant drift velocity for a given condition. The slope of the center-line decreases with an increase in wave length until it attains a negative value (Figure 25d). This indicates that the model is drifting against the direction of wave travel.

In Figure 26 the computed drift velocities are plotted as a function of the ratio of wave length to ship length. There is a scatter in the experimental points, but the trend seems to be well defined. The drift velocity seems to depend also on the shape of body as it is noted that its rectangular shape gives the highest velocities. The ship model seems to start to drift against the waves when the wave lengths exceed approximately 5 or 6 ship lengths.

Similar results of drift directions and velocities also have been obtained in Sweden [14]. In this instance the problem occurred in connection with a law suit resulting from the collision of a steamship (1,075 tons - 184 ft. long) and a tanker (16,700 tons - 465 ft. long). Both ships were anchored in the Great Belt S.W. of the Sjalland reef, Denmark. The crew of the tanker stated that their ship had remained in position and at anchor and that the steamship had been carried toward the tanker by the strong westerly wind and the high seas. The crew of the steamship were equally sure that their ship had remained at anchor, and that the tanker had been carried by the ocean current and dragged, against the westerly wind and against the waves, and had collided with the steamship. To yield information on the problem, tests were completed at the Hydraulic Laboratory, Stockholm.

A number of tests were carried out on models where the two ships were oriented parallel to the waves. A second series of tests were conducted in a flume using only the mid-section of the ship, as shown in the sketch below.



Here the model filled the width of the wave channel, except for a small space of a few millimeters so that the model should not touch the flume walls. The tests were carried out to a scale of 1:20.

The results were that the ship generally moved with the waves, but sometimes it stood still, and occasionally even moved against the waves. The direction of movement depended on the wave-length and on the vertical oscillation of the ship.

The comparison of the present results with those obtained in Sweden can not be given in this report as the beam of the ship is not given in the Swedish report. Approximate calculations, however, gave good agreement between the two results. The shape of the experimental curve obtained in Sweden is very similar to that in Figure 26.

V. COMPARISON OF THEORY WITH RESULTS

As mentioned before, the purpose of this report is primarily to describe the laboratory procedures and present some experimental results. Parallel to the laboratory studies, theoretical investigations were made by Fuchs and MacCamy and a new theory has been developed for ship motion in non-uniform waves. The theoretical background and the new theory, along with comparison with laboratory data results, are given in a separate report[2]. The results so far available indicate that the new theory predicts the ship motion for non-uniform long-crested waves very accurately.

Here, however, an attempt will be made to compare the results obtained in regular waves with an earlier theory introduced by Weinblum (1933)[7,15,16].

Weinblum made the following assumptions:

1. The waves are uniform sine waves, in form.
2. The pressure distribution in waves is not affected by the presence of the ship (an attempt has been made to use correction-coefficients for the hydrodynamic forces).
3. The ship is wall-sided.

This earlier theory has, however, been extended by Weinblum and St. Denis[9] resulting in a very accurate theory for long-crested uniform waves. In the extended theory the assumption that the ship is wall-sided is not essential.

The reason that the earlier theory of Weinblum has been used for comparison is to give a good idea of the mechanics of ship motion in uniform waves without going into comprehensive mathematical theory. A comprehensive theory is given by Fuchs and MacCamy in a separate report[2]. This theory is essentially very similar to the extended theory by Weinblum and St. Denis, but is extended to apply to non-uniform waves.

In this report the comparison of results with Weinblum's theory should be regarded more qualitative than quantitative and should show the trend of ship motion under different wave (uniform) conditions. This goal has been more than reached because the results show not only similar trends in theory and results, but also very close agreements quantitatively, when the assumptions made by Weinblum are fulfilled.

For the case in which the longitudinal axes of the ship is parallel to the

direction of wave travel we have the following differential equations:

$$\rho\, V_z'\,\frac{d^2 z}{dt^2} + 2N_1\frac{dz}{dt} + \rho g A z = \rho g r B L\, \mathcal{E}_{(\lambda/L)} \cos\omega t \tag{8}$$

for the heaving motion

and

$$I_y'\,\frac{d^2\Theta}{dt^2} + 2N_2\frac{d\Theta}{dt} + D_v\,\overline{M_L G}\,\Theta = \rho g r\,\frac{BL^2}{2}\,\vartheta_{(\lambda/L)} \sin\omega t \tag{9}$$

for the pitching motion.

These relationships give the solutions:

$$z = r\,\frac{\mathcal{E}_{(\lambda/L)}}{a_w}\,\mu_1 \cos(\omega_1 t - \varepsilon_1) \tag{10}$$

$$\Theta = \frac{r}{L}\,\frac{\theta_{(\lambda/L)}}{2C_L}\,\mu_2 \sin(\omega_2 t - \varepsilon_2) \tag{11}$$

From these equations it can be seen that amplitude of the heaving motion depends on two independent functions; that is,

1. Heaving force function $\mathcal{E}_{(\lambda/L)}$
2. The amplitude distortion function $\mu_1\,(\omega/\omega_1)$

The amplitude of the pitching motion depends also on the same kind of independent functions, that is,

1. Pitching moment function $\theta_{(\lambda/L)}$
2. Amplitude distortion function $\mu_2\,(\omega/\omega_2)$

<u>Amplitude Distortion Function:</u> Amplitude distortion functions μ_1 and μ_2 are governed principally by the ratio of the natural frequency ω_1 or ω_2 of the heaving or pitching oscillation to the frequency of excitation, but also by the degree of damping.

We have:

$$\mu_1 = \frac{1}{\sqrt{\left[1-(\omega/\omega_1)^2\right]^2 + (2w_1/\omega_1)^2\,(\omega/\omega_1)^2}} \tag{12}$$

$$\mu_2 = \frac{1}{\sqrt{\left[1-(\omega/\omega_2)^2\right]^2 + (2w_2/\omega_2)^2\,(\omega/\omega_2)^2}} \tag{13}$$

In these equations w_1 and w_2 represent damping values per unit mass. It is interesting to point out that $2w_1/\omega_1$ for the heaving and $2w_2/\omega_2$ for the pitching has been found in numerous model tests* to be of approximately an equal value

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* Completed in the "Versuchsanstalt für Wasserbau und Schiffbau, Berlin"

of 0.45[4]. This has been checked in previous experiments and was found to be true in the present tests on the ship model and cylinder wherein $\frac{2w_1}{\omega_1} = \frac{2w_2}{\omega_2} = 0.44$. For the rectangular block, it was not possible to obtain the same value from different experiments. But it was found also that $\frac{2w_1}{\omega_1}$ and $\frac{2w_2}{\omega_2}$ are of approximately the same magnitude and varied between 0.30 and 0.60 with an average value of 0.45. The reason for this variation seems to be that the rectangular block was too small and light as a model and various unpredicted outside conditions might have considerable effect on the damping curve. It can be concluded that models of very small size should not be used.

In Figure 29 $\mu_{1,2}$ has been plotted as a function of $\frac{\omega}{\omega_{1,2}}$ with $\frac{2w_{1,2}}{\omega_{1,2}}$ as a parameter. The curves have been plotted for values of $\frac{2w_{1,2}}{\omega_{1,2}} = 0.50$, 0.45 and 0.40. As can be seen from these curves, the variation in $\frac{2w_{1,2}}{\omega_{1,2}}$ may cause a considerable difference in heaving and pitching amplitudes in the region close to resonance, i.e. $\omega/\omega_1 = 1$. Outside of this region the difference would be relatively small. For comparison of theoretical and experimental results, $\frac{2w_{1,2}}{\omega_{1,2}} = 0.45$ is used in this report.

Heaving Force Function $\mathcal{E}(\lambda/L)$: The heaving force function, $\mathcal{E}(\lambda/L)$, depends primarily on the variable or quantity λ/L and the ship form. Tables have been completed to compute $\mathcal{E}(\lambda/L)$ for various ship forms (9;15). Figure 27 illustrates the shape of the heaving force function $\mathcal{E}(\lambda/L)$ for water lines $\alpha_w = 1$ (rectangular block) and $\alpha_w = 0.850$ (ship model). They may be interpreted as force curves for a given ship when the wave height is constant while the wave length varies. From this figure we see:

1. The heaving force function is zero when λ/L value is approximately equal to the water plane coefficient.

2. From item (1) the conclusion might be drawn that at an effective wave length equal to the ship's length the heaving force is moderate or may be even zero (rectangular waterline).

3. As the wave-length approaches infinity, the value of the heaving force function approaches asymptotically the value of the water plane coefficient.

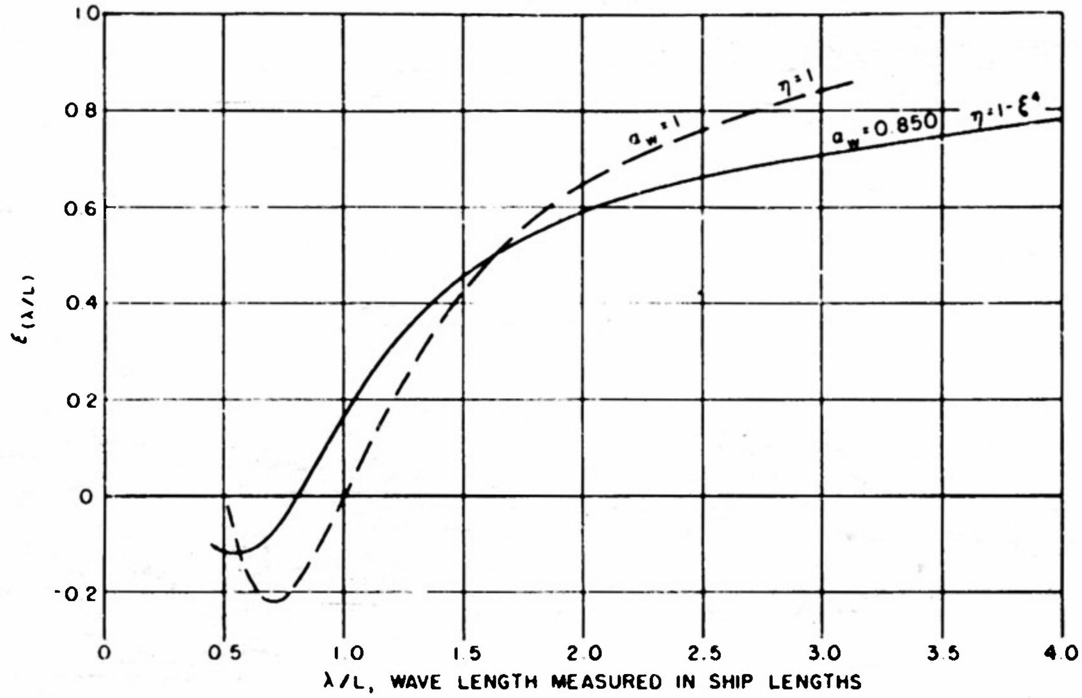
The Pitching Moment Function $\theta(\lambda/L)$: The pitching moment function, $\theta(\lambda/L)$, can also be obtained by using previously prepared tables (9; 15). Figure 28 gives us $\theta(\lambda/L)$ for values of water plane coefficients $\alpha_w = 1$ and $\alpha_w = 0.850$. As can be seen from these curves:

1. The pitching moment is small for short wave lengths, especially when the wave length is approximately 1/2 of the ship's length or less.

2. The pitching moments are high when the wave length is equal to the ship's length, with maximum being between λ/L values of 1 and 1.5.

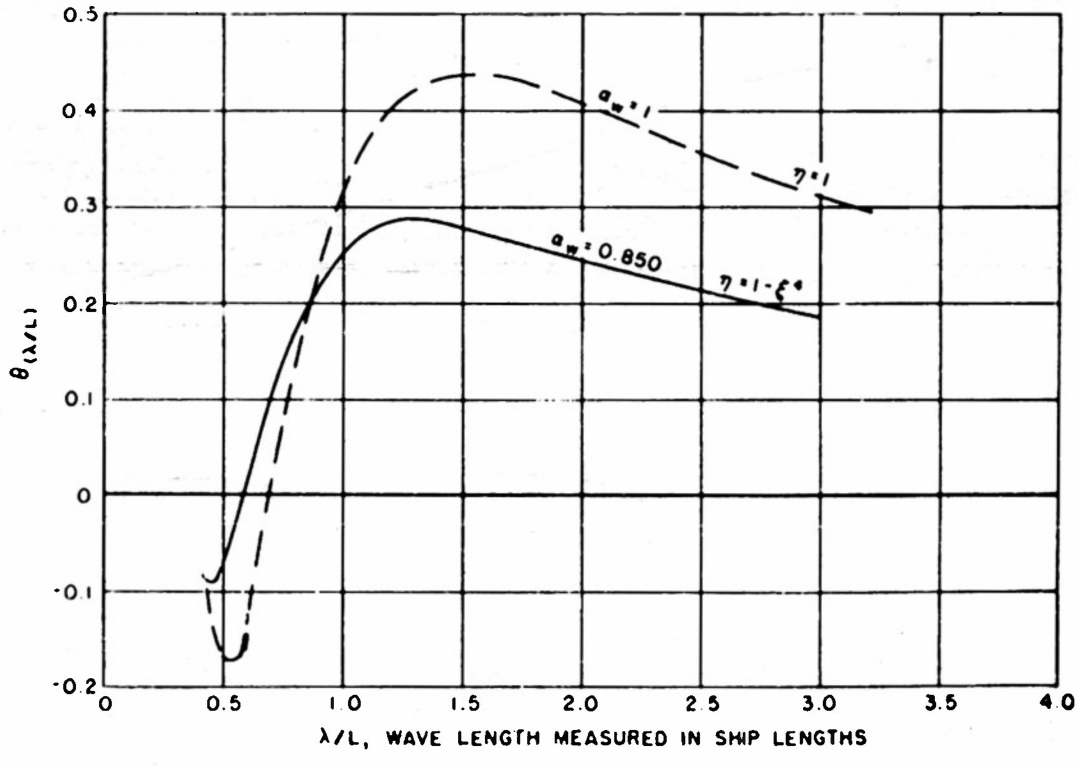
3. The occurrence of maximum pitching moment depends upon the ship form, especially upon the water plane coefficient, α_w. For fuller water lines the maximum pitching moment occurs at longer wave lengths than for slender ships.

4. The asymptotic value of the pitching moment is zero when the wave length approaches infinity. This is natural, for the slope of the wave disappears for infinitely long waves.



Heaving Force Function, $\varepsilon_{(\lambda/L)}$, for a rectangular waterline, $\eta = 1$, and for a parabolic waterline (ship model), $\eta = 1 - \xi^4$

FIG. 27



Pitching Moment Function, $\theta_{(\lambda/L)}$, for a rectangular waterline, $a_w = 1$ and $\eta = 1$, and for a parabolic waterline (ship model), $a_w = 0.850$ & $\eta = 1 - \xi^4$

FIG. 28

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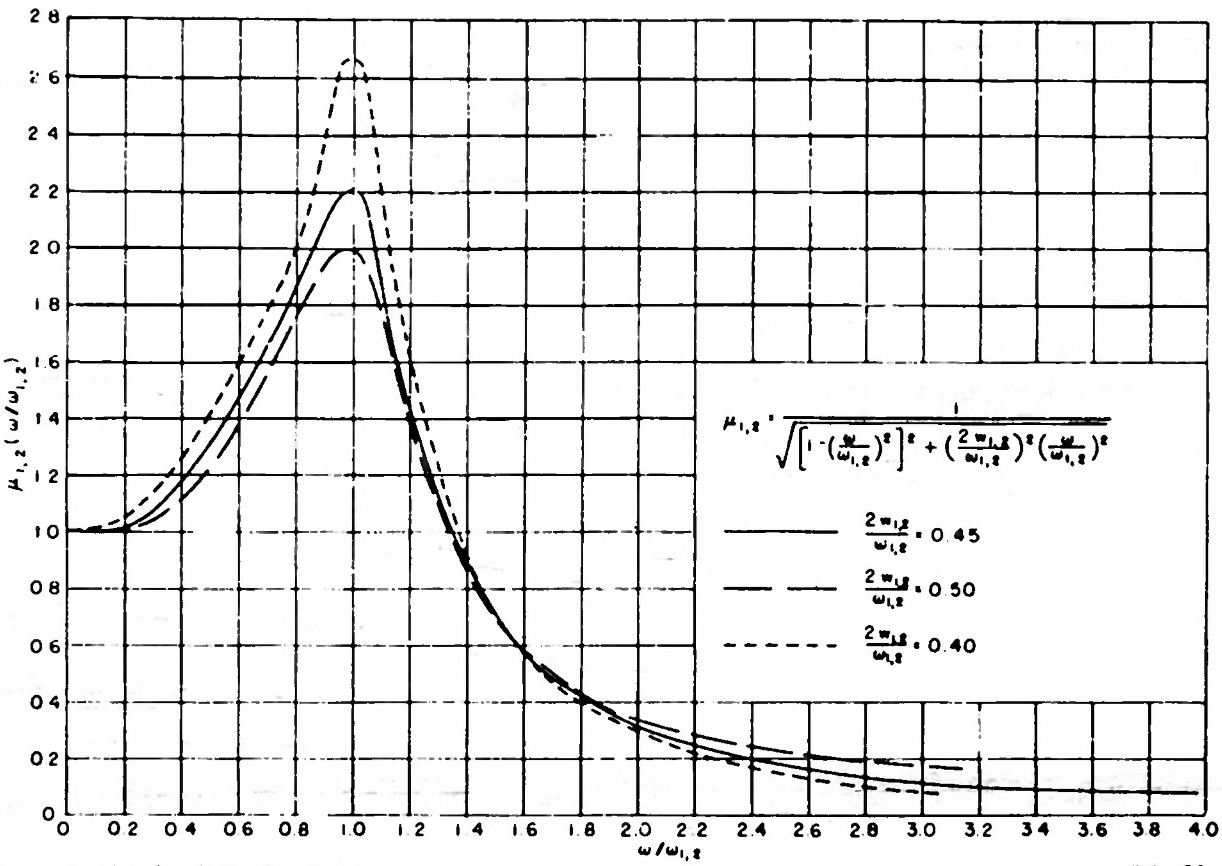


Amplitude-distortion factor, $\mu_{1,2}$

FIG. 29

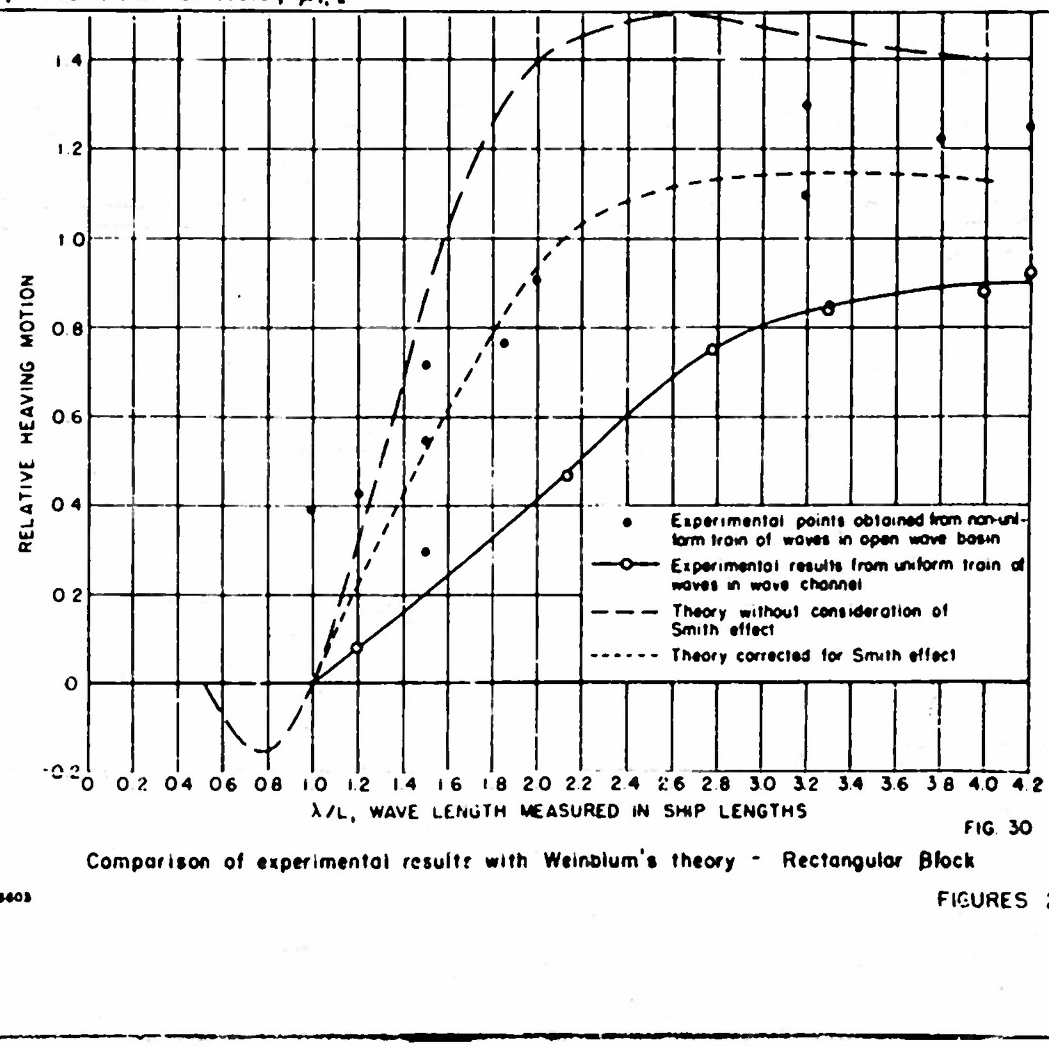


FIG. 30

Comparison of experimental results with Weinblum's theory - Rectangular Block

In general it might be pointed out that maximum amplitudes both for heaving and pitching need not occur at resonance, for the resulting amplitudes are functions of amplitude distortion factor μ, as well as of the exciting force. Hence, when the force factor is very small, the resulting amplitudes also will be small even when the wave period is equal to the natural period of oscillation of the ship $(\omega/\omega_1 = 1)$.

Comparison of Results: The results are given in Figures 30 through 34 and compared with the theory. In these graphs relative heaving amplitudes ζ/η (or relative pitching amplitudes Θ/H) are plotted as functions of relative wave lengths. Here ζ represents the heaving amplitude, η the amplitude of wave motion at the same moment, Θ the pitching angle in radians, and H the wave height that causes this pitching angle.

The theoretical curves are obtained from equations 10 and 11 by re-writing these equations and putting the heaving amplitude ζ, instead of z and the wave amplitude η instead of r.

Thus,

$$\frac{\zeta}{\eta} = \frac{\mathcal{E}_{(\lambda/L)}}{a_w}\mu_1 \tag{12}$$

$$\frac{\Theta}{H} = \frac{1}{2L}\,\frac{\theta_{(\lambda/L)}}{2C_L}\mu_2 \tag{13}$$

For the rectangular block $a_w = 1$ and $2C_L = 1/6$
For the ship model $a_w = 0.850$ and $2C_L = 1/10$

In these equations the Smith effect is not considered. It is known that the isobars within the wave itself are given by

$$r_z = r\,e^{-2\pi z/\lambda}\sin(kX - \omega t) \tag{14}$$

where r is the half wave height. Thus, for the ship motion study the effective wave heights should be used and not the geometrical wave height. The effective wave height is given by

$$r_{effect} = r\,e^{-2\pi d/\lambda}$$

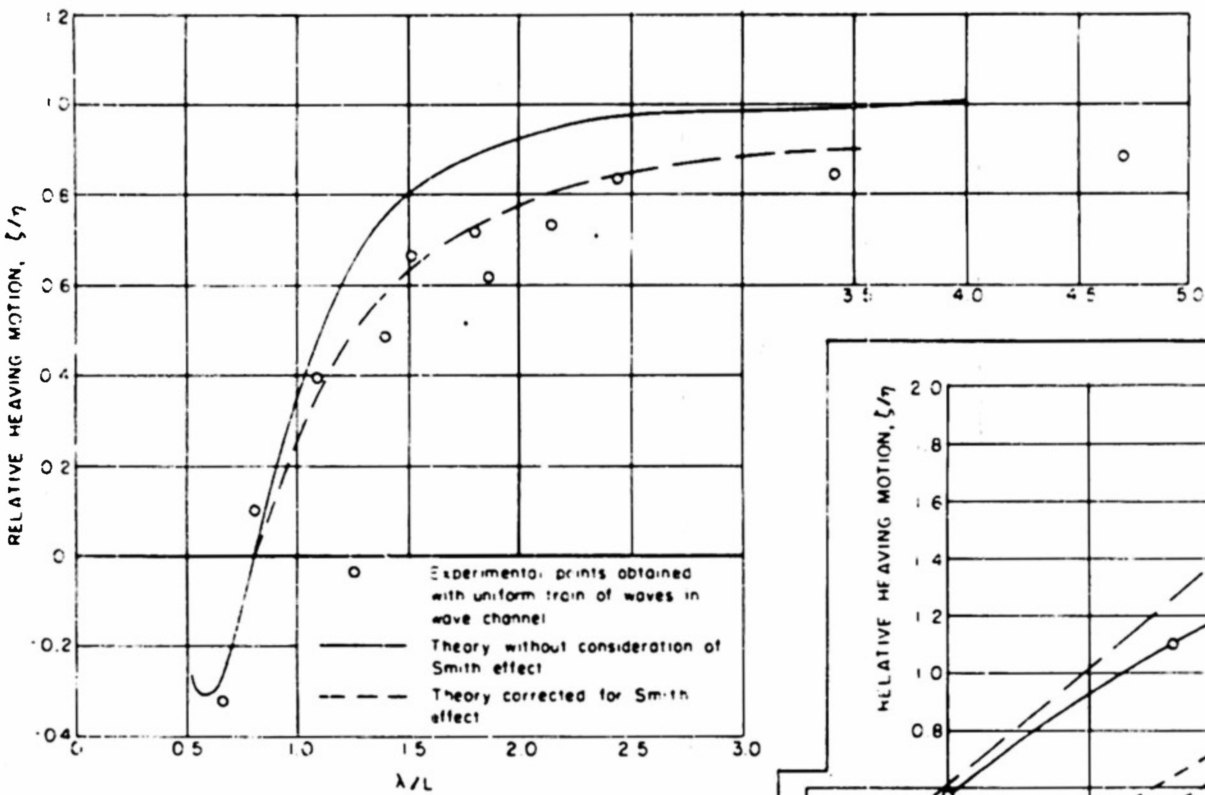
where d is the draft of the ship (true only for wall-sided ship with a flat bottom). In Figures 30 through 34 the theoretical curves are given for comparison for both of the following conditions:

(a) as computed by using geometrical wave heights,

(b) as computed by using effective wave heights.

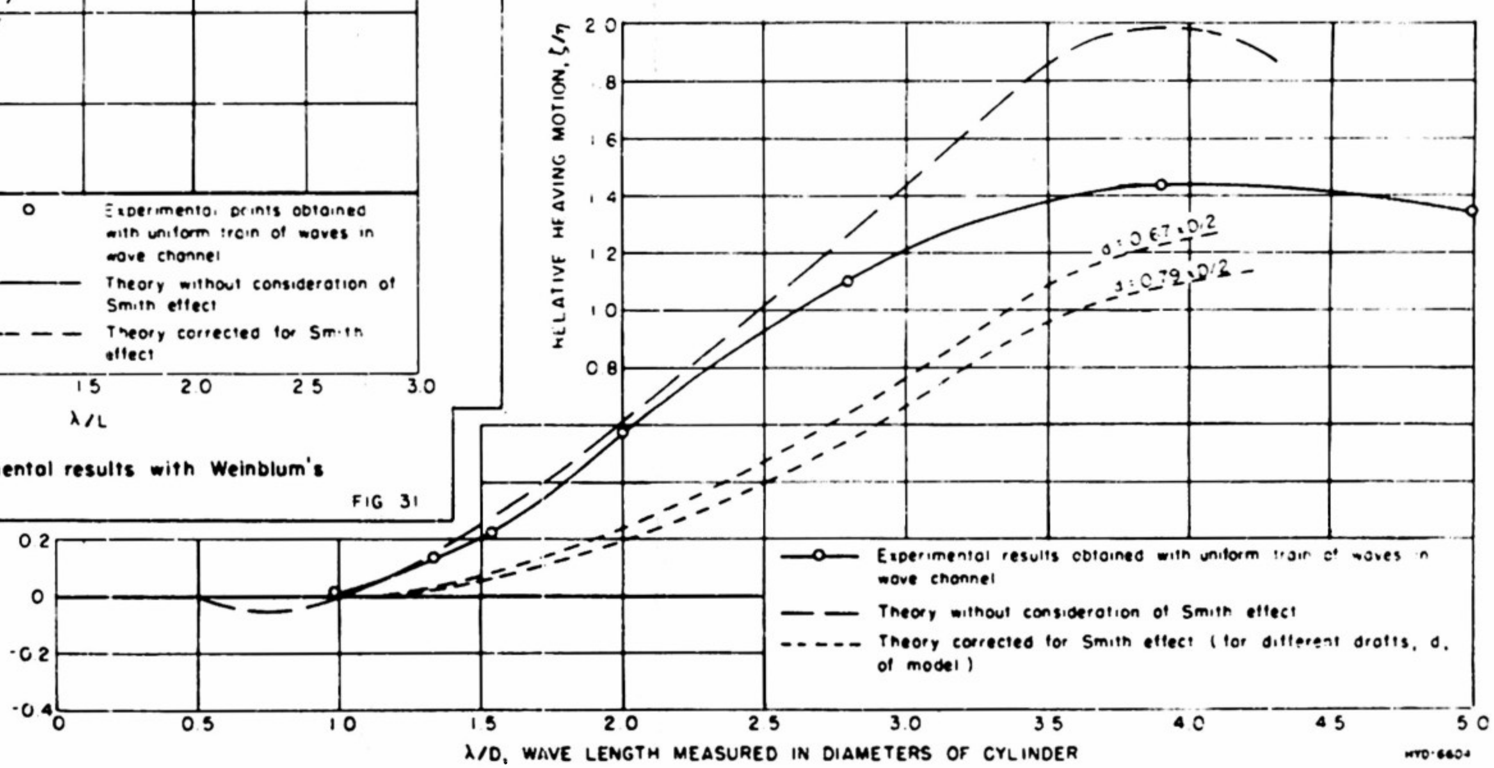
For all cases the curves computed using effective wave heights (corrected for Smith effect) give the best agreement with experimental data.

In Figure 30 the experimental results are compared with theory for the heaving of the rectangular block. The results obtained in an open basin with non-uniform waves agree well with the theory. The results obtained in the wave channel with regular waves are considerably lower but give a much smoother curve.



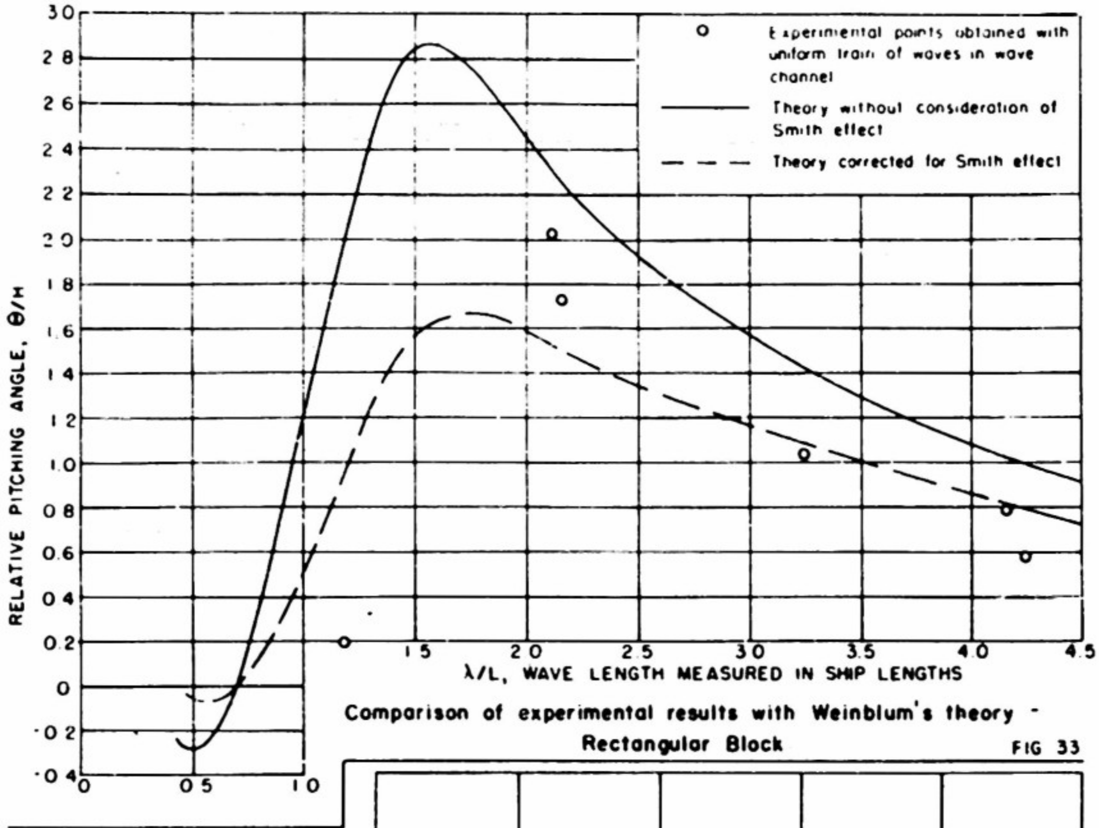
Comparison of experimental results with Weinblum's theory - Ship Model

FIG 31



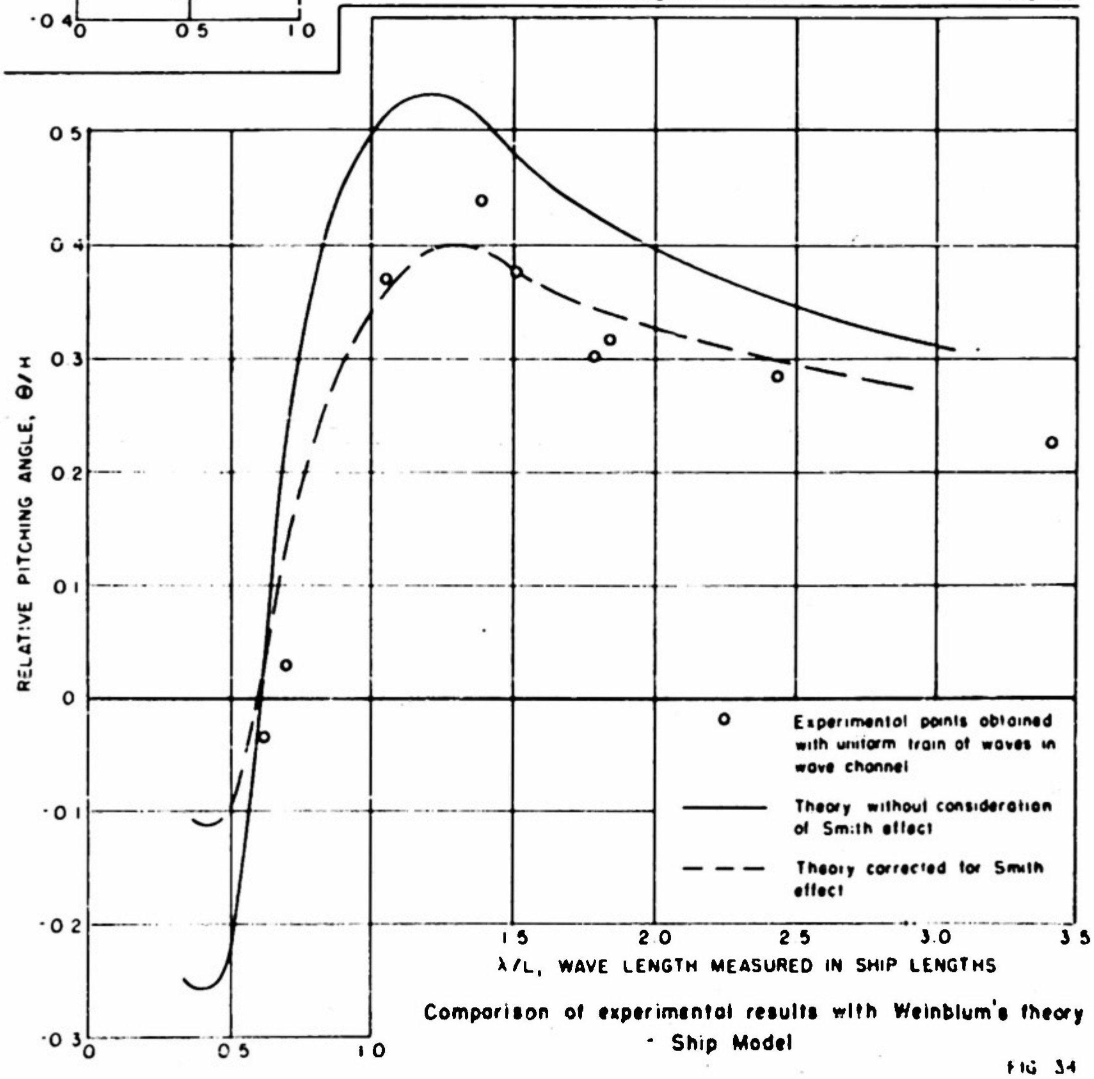
Comparison of experimental results with Weinblum's theory - Cylinder

FIG 32



Comparison of experimental results with Weinblum's theory - Rectangular Block

FIG 33



Comparison of experimental results with Weinblum's theory - Ship Model

FIG 34

In Figure 31 the experimental results as obtained with the ship model in uniform waves and in the wave channel are compared with the theory. The agreement can be considered as very good.

The ship model and the rectangular block are both wall-sided and agree with the assumptions made by Weinblum for his theory. To observe the conditions which occur when the model is not wall-sided, some experiments were completed with a circular cylinder, as previously described, and the results given in Figure 32. The experimental points fit a smooth curve very well, but do not agree with theoretical results as far as the shape of the curve and the absolute values are concerned. To compute the effective draft for a ship, Horn(4) gives us the following formula:

$$d_{eff} = V/A$$

where V is the displacement and A is the waterplane area. The effective draft for the cylinder was computed as $d_{eff} = 0.79\frac{D}{2}$, and the theoretical curve was corrected for this value (D is the cylinder diameter). Also another effective draft with $d = 0.67\frac{D}{2}$ was used, as shown in Figure 32. ($0.67\frac{D}{2}$ was derived by integrating the Smith effect over submerged area.) The experimental curve fits the uncorrected theoretical curve for the shorter wave lengths and then departs to fit the corrected curves for long waves. This demonstrates that Weinblum's theory may be used with good results when the ship is wall-sided, but gives values which are too low (when the curves are corrected for Smith effect) for parabolic ship cross-sections, especially for the shorter wave lengths.

In Figure 33 and 34 the experimental results of pitching angle are compared with the theory. The agreement is good again, especially for the experiments with a ship model.

VI. CONCLUSIONS

1. For long wave lengths as compared to ship length, the ship model follows the waves smoothly and the heaving amplitude is very close to the wave amplitude.

2. The heaving amplitude of the ship decreases with decreasing wave lengths and may become very small.

3. For short wave lengths a shift in phase between the wave motion and ship motion occurs resulting in slamming under some conditions.

4. The surging motion is much higher than is generally believed and should be taken into consideration in towing tests.

5. When the ship is not underway, and when it is not affected by winds or currents, it drifts in the direction of the waves for shorter wave lengths, stays still in longer wave lengths and may occasionally move against the direction of wave travel in very long waves.

6. A conclusion that follows, considering heaving, pitching, surging and drifting motions together, is that the ratio of wave length to ship length equal to unity does not necessarily represent extreme conditions for all of these motions. The extreme values depend very much on the characteristics and the shape of the ship and should be examined for each motion separately.

7. The experimental results of heaving and pitching motion agree with Weinblum's theory when the ship is wall-sided and the waves are uniform sine or cosine waves.

8. Although the experimental results agree with the theories, the conclusions should not be drawn that these theories are applicable for actual conditions in the ocean when short-crested waves are encountered.

APPENDIX

Relatively few prototype measurements are available under actual ocean conditions. One of the best set of experiments in this regard was the sea tests of M.S. "San Francisco" in 1934[4]. Fortunately different sea and wind conditions were encountered during this trip with winds ranging from calm to maximum values. The maximum wind force of 12th Beaufort scale was encountered on the North Atlantic. In Table I some characteristic values are given as obtained during these measurements to give some idea of the expected actual conditions and permit comparison with laboratory values.

TABLE I

Date	Wind Beauf. No.	Av. wave length Ft.	Av. wave height Ft.	Max. accel. midship ft/sec^2	Max. heav. midship ft.	Max. accel. at bow ft/sec^2.	Max. heav. at bow ft.	Max. Pitch. angle * degrees
Nov. 12,1934	3-6	230-260	13	4.6	13.5	9.7		10
Nov. 23	7	estimated 250	estimated 19.7	6.25	16.5			
Dec. 2	6	390	13 - 19.5	6.85	22.3	17.5	49	11
Dec. 9	5-6							
stormy days Dec. 11		820	49	4.9	41.4	11.5	82	17
Dec. 11		750-1500						
Maximum Values		750-1500	59**			14.8		Dec.11 24

* Total of negative and positive degrees

** The wave height obtained by using stereo-photogrammetric method and so should be reliable.

M.S. "SAN FRANCISCO" -	Displacement	13,073 T.
	Additional lading	7,624 T.
	Length	430 FT.
	Draft	24 FT.

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